E. Roy Weintraub, How Economics Became a Mathematical Science (Duke University Press, Durham and London, 2002).

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Mathematization processes have traditionally been an issue of special interest for historian of science of all sorts. Above all, the historically changing interrelations between physics and mathematics have provided fruitful material for a great amount of historical research over the last several decades. This particular topic has also provided the paradigmatic example of how quantitative methods have been adopted and variously used to study and formulate scientific theories. Fewer are the works devoted to similar studies on the mathematization of the social sciences, but works that have been published in this field clearly suggest that the issues involved here are no less historically appealing than those that arise in connection with the use of mathematics in the natural sciences. To the extent that they are interested at all in the historical development of their own academic discipline, the question of the mathematization of economics may naturally appear as one of special interest for economists and social scientists. For historians of science in general, and historians of mathematics in particular, this question also connects with additional, more general disciplinary concerns.

E. Roy Weintraub's new book is not the first to deal with the history of mathematical economics since the end of the nineteenth century. Two other well-known books that have become standard for anyone interested in learning about the topic can be mentioned in this context. In Philip Mirowski's *More Heat than Light* (1989) the development of neoclassical economics is presented as an evolving attempt to emulate, within the realm of the social sciences, the insights provided in physics by the energy conservation principle. Economic theory, in this account, would consider individuals in analogy with physical particles. Very roughly stated, whereas the motion of particles, unless affected by external forces, is determined by their potential energy, the economic analogy would imply that the individuals, acting under the constraints of a given budget, would move within a 'commodity space', their 'motion' being determined by the utility function. In analyzing how some leading economists since the late nineteenth century developed their ideas under the spell of this metaphor, Mirowski's account is not only descriptive, but also critical: in physical law, he stressed, energy conservation applies to the sum of potential and kinetic energies, and those who followed the energetic analogy in economics failed to adopt and implement the full picture, thus loosing the effectiveness of the idea in its original context. More recently, Mirowski extended his account to include, in a new book, the influence of information science on economic thought. In *Machine Dreams: Economic Becomes a Cyborg Science* (2002), he is no less critical of

the adoption of external metaphors as guiding principles in the elaboration of economic theories.

A second standard book on the topic is *The Invisible Hand* (Original Italian Version: 1987) by Bruna Ingrao and Giorgio Israel. This book describes in detail various stages in the development of general equilibrium analysis, by focusing on the adoption of mathematical ideas originating outside economics. According to Ingrao and Israel these ideas integrated into existing trends within European social thought that attempted to quantify the kind of phenomena under study. This integration of ideas originally arising in separate contexts soon came to dominate economic thought, while leaving all other, non-mathematical components outside it. Like Mirowski's, this account of modern economics is written from a rather critical perspective.

Weintraub's new book is both less and more ambitious than either of these two. It is less ambitious in the breadth of topics it encompasses, since it does not attempt to be as comprehensive as they do. It focuses on a specific group of prominent figures and events, while consciously leaving many others outside its discussion. It is more ambitious, however, in the attempted depth it aims to achieve, namely, a detailed account of the actual *process of interaction* and of the complex dialogue between the disciplines of mathematics and economic theory. Weintraub stresses that an adequate analysis of this kind must take into account an element that was essentially overlooked in previous, related works, namely the changing ways in which mathematicians conceive the nature and scope of their own discipline. These historically conditioned conceptions, rather than the specific results or techniques known to mathematicians at any given stage, are the main objects of attention that Weintraub considers in his own analysis and that allow him to pursue his line of historical research.

Since some of the basic ideas underlying Weintraub's methodology are admittedly taken from my own research on the history of mathematics, I will allow myself explaining these ideas briefly here. They are based on distinguishing between the 'body' and the 'images' associated with any field of scientific knowledge. The former includes statements pertaining to the subject matter *of* the discipline, whereas the latter comprise claims which express knowledge *about* the discipline. The body of knowledge includes theories, 'facts', methods, open problems. The images of knowledge serve as guiding principles, or selectors. They pose and resolve questions which arise from the body of knowledge, but which are in general not part of, and cannot be settled within, the body of knowledge itself. The images of knowledge determine which of the open problems of any discipline are more intensely addressed by its practitioners at a certain point in time. They also determine what is to be considered a relevant experiment, or a relevant argument in the discipline and what is to be taken as its legitimate methodology. Likewise, university curricula are built on the basis of dominating images of a discipline.

On the basis of this distinction my own research has been aimed at explaining certain important mathematical developments around the turn of the twentieth century, prominent among which is the rise of the structural approach that dominated algebra, and later on a considerable part of the overall mathematical activity all over the world, at least between 1930 and 1970 (Corry 1996, 2003). The underlying idea is that the historian of mathematics gains important insights by looking at changes in the body of mathematics against the background provided by the changes in the images of mathematics. In fact,

although significant mathematical breakthroughs usually come by way of new solutions to open problems, new techniques and theories developed in the way to those solutions, or new theorems and new proofs, many of the most important long-term changes in the discipline are those pertaining to the images of mathematics.

Weintraub found out that this scheme should not remain circumscribed to mathematics itself, but that it had a useful explanatory power for understanding the development of mathematical economics as well. He relied on it, in the first place, in order to explain the rise to prominence of Gerard Debreu and the process of reception, and widespread adoption of his work within mathematical economics. Weintraub's discussion of this topic appears in Chapter 4 of his book but he had also published an earlier version of it, together with Mirowski, in 1994. (In fact, some other earlier works were also elaborated into additional chapters of the book.)

The case of Debreu and the influence of Bourbakian mathematics on his thought best exemplifies the kind of argument put forward in Weintraub's intended overall analysis. Bourbaki was a pseudonym adopted by a group of young, French mathematicians who undertook in the mid-1930s the writing of a multi-volume treatise on mathematical analysis that would provide the new basis for university teaching in their country. With time, the members of this group individually became leading mathematicians, and the views put forward in their common work were widely adopted in the mathematical world. These views implied the pursuit of an extreme deductive rigor in the development of theories with little or no concessions—certainly not at the declarative level—to intuitive or graphical motivations, specific examples or external applications, and didactical considerations.

As Weintraub remarks in his book, the very idea of a Bourbakist incursion into a field of applied mathematics, and particularly economics, sounds as an oxymoron. And yet the work of Debreu in the postwar period provides precisely an instance of such an incursion, and a very successful and quickly adopted one at that. Weintraub analyzes this intriguing story by focusing on the images of mathematics of Bourbaki, rather than on specific results that could be gathered from their books, and by explaining how these images were successfully implanted into the realm of economic theory.

Bourbaki played a fundamental role in establishing the dominant images of mathematics between 1935 and 1980, and Weintraub analyzes how this dominance became crucial in the interaction of this discipline with the contemporary development of economics. Bourbaki consistently declared himself to be the intellectual heir of David Hilbert, one of the two most influential mathematicians (the other one was Henri Poincaré) of the beginning of the century. This claim no doubt helped establishing their authority, even though it distorted to a large extent some of the central traits of Hilbert's own images of mathematics (Corry 1998). The link between Hilbert and Bourbaki is usually established through the idea of "mathematical formalism", of which the two were allegedly prominent representatives. The situation, however, is much more complex. A thorough understanding of the different images of knowledge of Bourbaki and of Hilbert, especially concerning the role of axiomatization and of a putative formalist approach related with the latter, turns out to be central to important developments in twentieth-century mathematics. Weintraub shows in his book that this is also the case for the development of mathematical economics. In chapter 3 he relies on an account of Hilbert's

complex conceptions, such as I have provided by analyzing his images of mathematics, and uses it to critically examine existing accounts of Hilbert's possible influence on the formalist views that gradually became dominant in economic theory.

A similar kind of analysis is applied to the other episodes and economists that Weintraub include as part of his analysis. Thus, for instance, in chapter 1 he opens with a discussion of Alfred Marshall. As the leading economist in late nineteenth century England, Marshall sought to secure the autonomy of his discipline, but he did not think that this should be done by transforming it into a mathematical science. "Burn the mathematics" is the title of this chapter, an advise quoted from a letter of 1906 by Marshall in which he recommends that one should make efforts to "prevent people from using mathematics in cases in which the English language is a short as the mathematical" (p. 22). Weintraub explains this attitude in terms of the images of mathematics that Marshall had acquired in his student days at Cambridge. The discipline was conceived there as an eclectic collection of tricks that the student should master in order to successfully undergo the Tripos examination.

Weintraub contrasts this image of mathematics with those of the German Felix Klein or of the Italian Vito Volterra, for whom mathematical rigor was warranted not by any kind of formalistic perspective, but rather by measurement and by bringing mathematics as close as possible to the constrains imposed by physical evidence. Among Volterra's students was the American Griffith C. Evans, whose work is analyzed in chapter 2 against the background of Volterra's images of mathematics.

Another way in which the historiographical scheme underlying Weintraib's analysis is manifest is in the interesting and uneasy dialogues that the mathematician Cecil G. Phipps established with two fellow economists: Don Patinkin (chapter 5) and William Baumol (chapter 6). With the latter, Phipps argued about standards of publication of the mathematical proof of a theorem related with economic theory (specifically, the Arrow-Debreu equilibrium theorem). With the former he debated the proper kind of interrelations to be established between the two disciplines. In February 1950 Patinkin, visibly upset by the attempts of this mathematician to curtail his attempts to develop the theory within his own standards, wrote a letter that illuminatingly brings to the fore what is perhaps a typical economist's point of view in this regard. He thus wrote (p. 174):

> "Your general objective of surveying mathematical economic literature from a rigorous viewpoint is very commendable. However, to accomplish this objective it is necessary to understand the purposes for which the economist is using the mathematical analysis. This should not be misinterpreted as saying that economic reasoning will make incorrect mathematics correct. But it definitely is intended to imply that failure to understand the economic background of the problem under discussion may well lead ... to *mathematical* errors. I should imagine that a pure mathematician would hesitate to pronounce definitive judgment on a question in mathematical physics without first thoroughly investigating the physical conditions involved; I think the same should be true for mathematical economics. To reverse your dictum, writers on economic questions should be held responsible for an understanding of economic analysis."

The parallel established here between physics and economics in their relationship with mathematics raises many intriguing questions that Patinkin was obviously totally unaware of, but that Weintraub's analysis may help further pursue in an instructive way.

Before concluding the book with some general reflections about methodology, historiography and the path of his own career against the background of the foregoing discussions, Weintraub devotes one chapter to recounting the story of his father, the economist Sidney Weintraub. Weintraub is described as a talented microeconomist and enthusiastic teacher whose career developed right when mathematical economics was becoming increasingly fashionable. The story of Sidney Weintraub connects with the main body of the book mainly because of his own lack of a strong mathematical background, a situation that left him out of some mainstream developments of the profession. At the same time, however, he did establish an interesting dialogue, akin to those described in earlier chapters, with his mathematician brother Hal Weintraub. Roy Weintraub presents the reader with a touching and illuminating exercise in the difficult art of analyzing one's own family history within a much broader and distant train of abstract ideas.

Weintraub's book brings to the fore an interesting and well-documented story full of insight on the complex interplay among pure scientific ideas, images of these ideas, and no less than that, institutional and historical factors. Readers with different fields of interest will most likely approach in diverse ways the many aspects of this account and judge differently the relative significance of the various points of views, threads of ideas, and individual figures whose contributions are discussed here. As a historian of mathematics I have already pointed out my positive impression due to the insightful use, in an originally unintended context, of a scheme aimed at shedding some additional light on the development of mathematical ideas. To the extent that they look outside their own relatively circumscribed field of interests and into neighboring disciplines, historians of mathematics have so far tended to study more in detail the relations of their own discipline and the history of physics. Weintraub's book makes it clear that there is also much to learn from a similar, detailed study of the history of economics. But more in general this book will make interesting reading to readers with a broader scope of interest in history of science at large. Finally, though my own professional expertise cannot really help me judge how economists may read and receive an account like this one, I do believe that Weintraub's book embodies the kind of research that can positively help practitioners of the discipline realize the kind of insight to be expected by studying the history of their field when presented in a stimulating, well-researched and authoritative way as is the case here.

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