How Useful is the Term 'Modernism' for Understanding the History of Early Twentieth-Century Mathematics?

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DRAFT – NOT FOR QUOTATION

Contents

1. Introduction

Mathematicians and historians of mathematics will mostly agree in acknowledging the period roughly delimited by 1890 and 1930 as a special period of deep change in the discipline in all respects: new methodologies developed, new mathematical entities were investigated and concomitantly new sub-disciplines arose, the relationship between mathematics and its neighboring disciplines was transformed, the internal organization into sub-disciplines was completely reshaped, areas of research that were very important in the previous century receded into the background or were essentially forgotten, new philosophical conceptions were either implicitly espoused or explicitly discussed, etc. The meteoric rise of Göttingen to world predominance has come to epitomize the institutional dimension of the substantial changes undergone by the discipline in this period. At the same time, however, other centers, both in the German-speaking world (such as Berlin, Munich, Vienna, Hamburg) and outside it (Paris, Cambridge), were also transformed in significant ways. Parallel to this, the scientific leadership of David Hilbert in Göttingen has been taken to embody, both symbolically and contents-wise, the personal dimension of the spirit of the period, side by side with other prominent names such as Emmy Noether, Giuseppe Peano, or Felix Hausdorff. As a whole, there is widespread agreement that it makes sense to see this entire historical process as a process of "modernization" of some kind in the discipline and to refer to the mathematics of this period (as I will do as well in what follows) as "modern mathematics".

The same period of time is also widely acknowledged as one of deep transformations in the visual arts, in music, in architecture, and in literature. The thoroughgoing changes that affected many areas of artistic activity are often seen as a response to the sweeping processes of modernization affecting Western society at the time. Concurrently, the term “modernism” is generally accepted as referring to an all-encompassing trend of highly innovative aesthetic conceptions typical of this “modern” era, broadly characterized by an unprecedented radical break with the traditions of the past in each area of cultural expression.

The influences of the new scientific ideas of the time (and particularly the influence of ideas arising in modern mathematics) on “modernism” in general, have received some attention by cultural historians trying to make sense of developments in their own fields of interest. By contrast, the question about the possibility of understanding the rise and development of “modern mathematics” as a specific manifestation of the broader cultural trend, “modernism” has not been seriously addressed as part of the mainstream history of the movement. Questions of this kind, however, have been recently pursued by a few historians of mathematics, and they still remain a matter of debate and a challenge for further research: Does it make historical sense to describe some or all of mathematics in this period, i.e., of “modern mathematics”, as a “modernist” project, in a sense similar to that accepted for other contemporary artistic or cultural manifestations? Does any such description help sharpening our historical understanding of mathematics as an intellectual undertaking with its own agenda, methodologies, and aims, but with deep connections to other fields of cultural activity? And can, on the other hand, an analysis of the history of mathematics of this period be of help to researchers investigating the phenomenon of modernism in other disciplines?

This paper is a programmatic attempt to discuss the conditions for a proper analysis of questions of this kind. In its basic approach, the paper is critical and indeed negative about the prospects of such an analysis, as it seeks to pinpoint the essential difficulties and potential pitfalls involved in it. In its underlying purpose, however, the paper is essentially positive, as it stresses the potential gains of such an analysis if properly undertaken. The paper also attempts to indicate possible, specific directions in which this analysis might be profitably undertaken. The main pitfall against which the paper wants to call attention is that of shooting the arrow and then tracing a bull's-eye around it. Indeed, one of the main difficulties affecting discussions of "modernism" in general (not just concerning the history of mathematics) is that of finding the proper definition of the concept, to begin with. One might easily start by finding a definition that can be made to fit the developments of mathematics in the relevant period just in order to be able to put together all what we have learnt from historical research and thus affirm that, yes, "modernism" characterizes mathematics as it characterizes other contemporary cultural manifestations. Although this approach has some interest, it does not seem to be in itself very illuminating, and indeed it runs the risk of being misleading since, by its very nature, it may force us to being unnecessarily "flexible" in our approach to the historical facts so as to make them fit the desired definition.

This article opens with an overview of some prominent ways in which the term "modernism" has been used in the historiography of the arts, and calls attention to some debates surrounding its usefulness in that context. This is followed by a discussion of three concrete examples of works that investigate the relationship between modernism in general and the modern exact sciences: on the one hand, an investigation of the influence of scientific ideas on modern visual arts (in the writings of Linda Henderson), and, on the other hand, two books (by Herbert Mehrtens and Jeremy Gray) that explore the connections of modern mathematics with more general, modernist cultural trends. In sections 4 and 5 I take two examples of authors discussing the roots and developments of modernist ideas in specific contexts (modernist painting in the writings of Clement Greenberg and Viennese modernism in a book by Allan Janik and Stephen Toulmin), and examine the possible convenience of using their perspective in discussing modernism and mathematics.

Beside the critical examination of some existing debates, on the positive side, a main point to be discussed in this article is that a fruitful analysis of the phenomenon of modernism in mathematics must focus not on the *common features* of mathematics and other contemporary cultural trends (including other scientific disciplines – mainly physics), but rather on the *common historical processes* that led to the dominant approaches in all fields in the period of time we are investigating. To the extent that the existence of what is described as common, modernist *features* in the sciences and in the arts has been explained in the existing literature, this has been typically done in terms of "Zeitgeist" or "common cultural values". Though useful at first sight, such an approach is, in my view, far from satisfactory because it actually begs the question. In contrast, a clearer understanding of the *processes* leading to the rise of modernism in certain intellectual fields may help us look for similar historical processes in mathematics that may have been overlooked so far by historians. If properly pursued, this might amount, in my view, to a significant contribution to the historiography of the discipline. Likewise, and no less interestingly, a clearer understanding of the historical processes that led to a putative modernist mathematics might shed new light on the essence and origins of modernism in general.

2. Modernism: A Useful Historiographical Category?

In trying to address the question of the possible usefulness of "modernism" as a relevant historiographical category for mathematics, the first difficulty to consider is that, in spite of its ubiquity, the fruitfulness of this concept in the context of general cultural history is far from being self-evident or agreed upon, and indeed its very meaning is still a matter of debate. "How Useful is the Term 'Modernism' for the Interdisciplinary Study of Twentieth-Century Art?" asked Ulrich Weisstein in an article of 1995, whose title I obviously appropriated for my own one here (1995). Weisstein's basic assumption was that the idea of "modernism" has indeed been used in fruitful ways in his own field of research, comparative literature, and from this perspective his question referred to its possible usefulness in relation with other fields, namely the visual arts and music. Of course, visual arts and music are fields of cultural activity which other scholars dealing with this period would certainly refer to as emblematic of modernism, and these scholars would perhaps ask about the usefulness of the term in literary research. At any rate, in answering his question Weisstein characterized modernism in terms of three basic features: (1) an emphasis on the formal, as opposed to thematic values; (2) an aesthetic compatible with the notion of classicism; and (3) a rite of passage through avant-garde.

This list of features surely has both merits and drawbacks for anyone trying to come to terms with the phenomenon of modernism. But again, other authors have proposed their own characterizations – some of them better known and often more often referred to than Weisstein's – which partially overlap with and partially differ from his as well as from each other's.[1]

Another example of an attempt to characterize modernism in terms of a basic list of features – a recent one that received considerable attention given the prominence of its author – is the one found in Peter Gay's *Modernism: The Lure of Heresy from Baudelaire to Beckett and Beyond* (Gay 2007). For Gay, this "lure of heresy" is part of a more general opposition to "conventional sensibilities" that characterizes modernism and can be reduced to just two main traits: (1) the desire to offend tradition and (2) the wish to explore subjective experience. All other features typically associated with the movement (anti-authoritarianism, abstraction in art, functionality in architecture, "a commitment to a principled self-scrutiny," and others) are for him derivative of these two. Indeed, "the one thing all modernists had indisputably in common was the conviction that the untried is markedly superior to the familiar, the rare to the ordinary, the experimental to the routine." Coming from a historian of Gay's caliber, this characterization seems rather unenlightening (and in a sense his entire book, though informative and rather comprehensive in its scope is – for me – frustratingly unenlightening), but this is not the place to come up with a detailed criticism of it. Rather, my point is to figure out whether, and to what extent, Gay's characterization (or any alternative one of this kind) might be taken as a starting point for assessing the question of modernism and the history of modern mathematics by going through the checklist it puts forward. This might involve a somewhat illuminating historiographical exercise, but it would also run the risk mentioned above in the introduction, namely that the checklist would provide a mould, or perhaps even a Procrustean bed, into which we would force the historical facts, even at the cost of distortion, either admitted or unnoticed. But even if the facts are not forced, the main shortcoming of this approach would still be, in my view, that little new light would be shed on our understanding of the historical material. "Modernism", I contend,

[1] Some well-known attempts to characterize modernism in various realms include (Calinescu 1987), (Childs 2000), (Eysteinsson 1992).

would become a truly useful historiographical category if it helped interpreting the known in historical evidence in innovative ways or, even, if it would lead us to consider new kinds of materials thus far ignored, or underestimated, as part of historical research on the development of mathematics.

In order to further clarify this contention, it seems useful to provide a brief description of the main trends of research in the history of modern mathematics over the last twenty-five years or so, and how these trends have tried, with various degrees of success, to take advantage and inspiration of historiographical ideas originating in neighboring fields. Research of this kind has certainly helped sharpen our understanding of the period that interest us here and of the complex processes it has involved. It has yield a very fine-tuned and nuanced view of the intricacies of historical processes in mathematics in general, and in "modern" mathematics in particular. And the question of modernism in mathematics has been in the background of many of these investigations, albeit more often implicitly than explicitly. Thus, we have learned much about the importance of local traditions and cross-institutional communication, and we possess detailed accounts of the development of specific, leading schools of mathematics in various countries.[2] We realize the impact of global events on the shaping of mathematical communities and trends.[3] We have come to historicize and to distinguish among varieties of ideas which are central to the mathematics of the period and which were previously taken essentially at face value and assumed to be fully and universally understood. This is the case with concepts such as "abstraction", "formalism", "certainty", "structuralism", and "set-theoretic",[4] as well as with the various threads involved in the so-called foundationalist debate of the 1920s.[5]

In the same vein, we have come to understand the specific contributions of individual mathematicians—leading figures and less prominent ones alike—and how their own

[2] See, e,g., (Begehr et al. 1998), (Biermann 1988), (Delone 2005), (Gispert 1991), (Goldstein et al. 1996), (Parshall & Rowe 1994) (Parshall & Rowe 1994), (Warwick 2003). The various works mentioned in this and in the next few notes are not intended as a comprehensive list, but rather as a representative one.

[3] (Kjeldsen 2000); (Parshall & Rice 2002); (Siegmund-Schultze 2001; 2009).

[4] (Corry 2004b); (Ferreirós 2007).

[5] (Grattan-Guinness 2000); (Hesseling 2003); (Sieg 1999).

careers and specific choices and styles influenced, and at the same time were influenced by, the more encompassing processes in the discipline.[6] We understand more precisely the processes that brought about the rise and fall of individual sub-disciplines and the mutual interaction across many of them,[7] as well as the role of specific open problems or conjectures in these processes.[8] We have a deeper understanding of the processes that led to the quest for full autonomy—and even segregation—as a central trait of "modernization" in mathematics,[9] but at the same time, we have a deeper knowledge of the substantial interaction and mutual influence between mathematics and its neighboring disciplines (mainly physics[10] and philosophy[11], but also economics[12] and biology[13]) during this period of time.

An important element recognizable in all of this recent progress in historical research of early twentieth-century mathematics is a sustained exploration of the inherent plausibility and possible usefulness of adopting historiographical categories and conceptualization schemes previously applied in related scholarly fields, and mainly in the historiography of other scientific disciplines. This started in the late seventies in relation with concepts such as "revolutions" and "paradigms" (Kuhn),[14] "scientific research programs" (Lakatos),[15] and with ideas taken from the sociology of knowledge[16] which, in an extreme version, led to the so-called "strong program" (David Bloor).[17] More recently it has comprised the reliance on ideas such as "research schools",[18] "traditions",[19] "images

[6] (Beaney 1996); (Corry 2004a); (Scholz 2001b); (van Dalen 1999); (Fenster 1998); (Curtis 1999).

[7] (Epple 1999); (Hawkins 2000); (Jahnke 2003); (James 1999); (Wussing 1984).

[8] (Barrow-Green 1996); (Corry 2010); (Moore 1982); (Sinaceur 1991); (Schappacher 1998).

[9] (Pyenson 1983).

[10] (Corry 2004b); (Lützen 2005); (Rowe 2001); (Scholz 2001a).

[11] (Peckhaus 1990).

[12] (Ingrao & Israel 1990); (Mirowski 1991); (Weintraub 2002).

[13](Israel & Millán Gasca 2002).

[14] (Gillies 1992).

[15] (Hallett 1979a; 1979b).

[16] (MacKenzie 1993).

[17] (Bloor 1991).

[18] (Parshall 2004).

of science”,[20] “epistemic configurations”,[21] “material culture of science”,[22] quantitative analyses,[23] and some others. In fact, such analytic categories have been sometimes adopted in a very explicit way and their usefulness has been both argued for and criticized. But at the same time, they have also entered the historiographical discourse of mathematics in more subtle ways and have been tacitly absorbed as an organic part of it. In one way or another, when historians of mathematics have made recourse to these analytic categories they have done it with a two-fold motivation in mind: (1) to broaden the perspectives from which the history of mathematics can be better understood, and (2) to broaden the perspective from which to understand the analytic categories themselves by examining a further, rather unique, domain of possible application. In both cases, historians of mathematics are anxious to establish bridges that may allow communication with neighboring disciplines (mainly history of science in general and philosophy of mathematics) and help overcome the essential “professional solipsism” that typically affects their scholarly discipline.

Seen against this background the question whether modernism may provide a useful category for understanding the history of mathematics at the turn of the twentieth century is both a specific manifestation of a broader trend in the historiography of mathematics and a leading theme that is pervasive throughout various aspects of this historiography. At the same time, however, when discussing modernism in mathematics as part of a more global intellectual process, some basic specificities of mathematics have to be kept in mind. Thus, in the first place, there are the essential differences between mathematics, on the one hand, and literature, art, or music, on the other hand. This is of course a much contended and discussed topic, and it is differently approached by various authors. In my discussion here, however, I will not go into any nuances, and I will take a clear stand in stressing this differences. I will consider mathematics to be a cognitive system definitely

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[19] (Rowe 2004a).

[20] (Bottazini & Dahan-Dalmedico 2001).

[21] (Epple 2004).

[22] (Galison 2004). Although more naturally classified as history of physics, this book devotes considerable attention to Poincaré’s mathematics as well.

[23] (Goldstein 1999); (Wagner-Döbler & Berg 1993).

involving a quest for objective truth, an objective truth that in the long run is also cumulatively and steadily expanding. Art, literature and music I will consider, on the other hand, as endeavors of a different kind, whose basic aims and guiding principles are different from those of mathematics and indeed of science in general. Considerations of objectivity, universality, testability, and the like, if they do appear at all as part of the aims of the artists or of the audiences, they appear in ways that differ sensibly from those of science. As pointed above, historians of mathematics are increasingly attentive to aspects of practice in the discipline that involve institutions, fashions and local traditions, but this does not imply an assumption that these aspects apply identically to mathematics as they apply to other manifestations of contemporary culture. This clear distinction, which I take as previous to and independent of the topic of this article, makes the possible application of the idea of modernism in mathematics, I think, all the more interesting and challenging, but also perhaps more implausible.

One consequence of particular interest of this distinction is that, whereas the possibility that mathematics may influence those other domains in all of their manifestations is a rather straightforward matter, the possibility of an influence in the opposite direction is a much more subtle and questionable one. I am not claiming that this latter kind of influence is altogether impossible. It may indeed manifest itself in various aspects of mathematical practice, in ways that would require some more space to specify in detail than I have available here. Schematically stated, external cultural factors can certainly influence the shaping of the "images of mathematics", namely, the set of normative and methodological assumptions *about* the contents of mathematics that guide the practice of individuals and collectives in mathematics, and help guide their choices of open problems to be addressed, general approaches to be followed, curriculum design, and the like.[24] These are all, of course, central factors in the development of mathematical knowledge, and they will directly affect the way in which the body of mathematics will continue to evolve. But these same cultural factors cannot directly alter the objectively determined truth or falsity of specific mathematical results. The objectively established truth of a result will not be changed in the future (except if an error is found – and this of course

[24] For a recent, particularly interesting example see (Graham & Kantor 2009).

happens). The importance attributed to the result may change, the way it relates to other existing results may change, but its status as an established mathematical truth will not change.[25] Herein lays a significant difference between mathematics and other cultural manifestations that interest us here (including other sciences), and this should be taken into account as part of our discussion of modernism.

A second, related consequence is the different relationship that each of these domains entertains with its own past and history. Many definitions of modernism put at their focus the idea of a "radical break with past". Such definitions will of necessity apply in sensibly different ways to the arts than to mathematics. Indeed, being guided above all by the need to solve problems and to develop mathematical theories, always working within the constraints posed by this quest after objective truth, the kinds and the breadth of choices available to a mathematician (and in particular, choices that may lead to "breaks with the past") are much more reduced and more clearly constrained than those available to the artist. In shaping her artistic self-identity and in defining her artistic agenda a modernist artist can choose to ignore, and even to oppose, any aspect of traditional aesthetics and craftsmanship. This implies taking professional risks, of course, especially when it comes to artists in the beginning of their careers, but it can certainly be done and it was done by the prominent modernists. The meaning of "a radical break with the past" in the context of mathematics would be something very different, and the choices open before a mathematician intent on making such a break while remaining part of the mathematical community are much more reduced. A mathematician cannot decide to ignore, say, logic (though she may suggest modifications in what should count as logic). Likewise, a mathematician cannot give up "mathematical craftsmanship" (if I am allowed this "abuse of language") as a central trait of the discipline, or as part of his own professional self-identity. That this is indeed the case derives, in the first place, from the essence of the subject matter of the discipline of mathematics. But at the same time it also derives from the peculiar sociology of the profession. An artist might decide to develop her own work and career by innovating within the field to an extent that cuts all connection with the contemporary mainstream in the relevant community (perhaps one must qualify this

[25] For a detailed discussion of the body/images of mathematics scheme and its historiographical significance see Corry 1989.

claim by adding that this is true after the turn of the twentieth century in ways that was not the case earlier than that). But such a possibility will simply *not* work in the mathematical profession, even for innovations that are not truly radical. In order to become a professional mathematician of any kind, one must first fully assume the main guidelines of the professional ethos.

The most prominent example that illustrates this point about the processes of professional socialization in mathematics is that of Luitzen J.E. Brouwer (1881-1966). Brouwer's 1907 dissertation comprised an original contribution to the contemporary debates on the foundations of mathematics. His thesis advisor urged him to delete the more philosophical and controversial parts of the dissertation and to focus on the more mainstream aspects of mathematics that it contained. This would be the right way, the advisor argued, to entrench the young mathematician's professional reputation and to allow developing an academic career to begin with. Brouwer's personality was undoubtedly one of marked intellectual independence and this trait was clearly manifest even at this early stage. Nevertheless, he finally came to understand how wise it would be to follow this particular piece of advice and he acted accordingly. It was only somewhat later, as he became a respected practitioner of a mainstream mathematical domain, that he started publishing and promoting his philosophical ideas, and to devote his time and energies to developing his new kind of radical, "intuitionistic" mathematics.[26] In addition, an important episode in the history of modern mathematics was the attempt of Brouwer to promote a different kind of logic, later called "intuitionistic logic". This was not meant as a call to abandon logic, or to make a radical break with the past, but rather to revert logic to a previous stage in its evolution, where no considerations of the actual infinite had (wrongfully and dangerously, from his perspective) made deep headway into mainstream mathematics. In this way, Brouwer intended to entrench the validity of logic rather than to innovate it in a radical manner.

A second issue that one must keep in mind in this discussion concerns the relationship between mathematics and other scientific disciplines, particularly physics. In terms of the

[26] See (van Dalen 1999, pp.89–99). On the question of Brouwer and modernism in mathematician see the contribution of José Ferreirós to the collection.

distinction stressed in the previous paragraphs natural science and physics fall squarely on the side of mathematics, as opposed to that of literature and the arts. And yet, in general terms, but with particular significance for the topic considered here, there are important differences with mathematics that should be taken into consideration throughout.[27] Thus for instance, a noticeable tendency among authors who undertake the question of modernism in science and in the arts is to include the theory of relativity in their studies as a fundamental bridge across domains.[28] There are, of course, many immediate reasons for this kind of interest raised by relativity (and in indeed, for explaining why this particular theory, and the persona of Einstein, attracts so much attention in so many different contexts), but I think that the ubiquity of relativity also helps clarify the relationship between mathematics and physics as it concerns the question of modernism. Thus for instance, when the theory of relativity is presented as a paradigm of modernist physics, one should notice to what extent this is claimed on the basis of its essentially physical contents and to what extent the peculiarities of its relationship with mathematics play role in this assertion.[29]

Of particular importance in this regard is what I have called elsewhere the “reflexive character of mathematical knowledge”. By this I mean the ability of mathematics to investigate some aspects of mathematical knowledge, *qua* system of knowledge, with the tools offered by mathematics itself.[30] Thus, entire mathematical disciplines that arose in the early twentieth century are devoted to this kind of quest: proof theory, complexity theory, category theory, etc. All of these say something about the discipline of mathematics and about that body of knowledge called mathematics, and they say it with the help of tools provided by the discipline, and with the same degree of precision and clarity that is typical, and indeed unique, to this discipline. Gödel’s theorems, for

[27] In an illuminating article about the uses of the terms “classical” and “modern” by physicists in the early twentieth century, Staley 2005, addresses this difference from an interesting perspective. In his opinion, whereas in physics discussions about “classical” theories and their status was more significant for the consolidation and propagation of new theories and approaches than any invocation of "modernity", in mathematics, different views about “modernity” were central to many debates within the mathematical community.

[28] See, e.g., (Miller 2000; 2001); (Vargish & Mook 1999).

[29] Constraints of space do not allow to elaborate this point here, but see (Rowe 2004b).

[30] (Corry 1989).

instance, are the paradigmatic example of results about (the limitations of) mathematical knowledge which were attained with tools and methods of standard mathematical reasoning and which therefore bear the forcefulness and certainty of any other piece of mathematical knowledge. It is true that also literature can become the subject matter of a literary piece, painting can become the subject matter of painting, and so on for other artistic endeavors. But as already stressed, whatever these disciplines can express about themselves, it will be different in essence of what mathematics can say about itself. On the other hand, exact and natural sciences other than mathematics cannot become the subject matter of research of themselves. Thus for instance, while mathematical proofs are tools of mathematics and the idea of “mathematical proof” can become the subject of mathematical research, “physical theory” or “biological theory” or “experiment” cannot become subjects of either physical or biological research. This unique feature of mathematics is not just interesting in itself, but it is also specifically relevant to the discussion of modernism, given the fact that the reflexive study of the language and methods of specific fields has very often been taken to be a hallmark of modernism in the arts, and that this reflexive ability of mathematics became so prominently developed in the period that interests us here.

The differences arising in this complex, triangular relationship between mathematics, on the one hand, the natural sciences on the other hand, and the arts in the remaining vertex, have been either implicitly overlooked or willingly dismissed in certain texts devoted to discussing modernism and the sciences.[31] It is my contention in this article that any serious analysis of mathematics and modernism must take them into account and stress them explicitly and consistently. A possible fourth vertex of comparison could include philosophy and the social sciences with their own specificities, but for reasons of space I will leave them outside the scope of the present discussion.[32] At any rate, what is of special interest for us here concerning these differences is to acknowledge their own historically conditioned character. In other words, whatever one may want to say about modernism in mathematics and its relationship with modernism in other fields, one must

[31] For a related discussion of this triangular relationship as related to narrative strategies in literary fiction, mathematics and history of science, see (Corry 2007).

[32] See (Ross 1994).

remember that the changing relationship among the fields must be taken to be part of this historical phenomenon.

3. Modern Mathematics and Modernist Art

I move now to examine some existing works that have explicitly addressed the connections between mathematics and the arts in the period 1890-1930, and to comment on them against the background of the ideas discussed in the previous section. First I focus on an analysis the possible influences of mathematics and the sciences on the arts, and then I move to consider the opposite direction.

An outstanding example of analyzing the influence of physics and mathematics on modern art in the early twentieth century appears in the work of Linda Henderson.[33] In her detailed scholarly research, Henderson has explored the ways in which certain scientific ideas that dominated the public imagination at the turn of the century provided "the armature of the cultural matrix that stimulated the imaginations of modern artists and writers" (Linda D. Henderson 2004, p.458). Artists who felt the inadequacy of current artistic language to express the complexity of new realities recently uncovered by science (or increasingly perceived by public imagination) were pushed into pursuing new directions of expression, hence contributing to the creation of a new artistic language, the modernist language of art. But in showing this, Henderson also studiously undermined the all-too-easy, and often repeated image of a putative convergence of modern art and modern science at the turn of twentieth century in the emblematic personae and personalities of Picasso and Einstein.[34] Contrary to a conception first broadly and famously promoted in Sigfried Giedion's *Space, Time and Architecture* (Giedion 1941), Einstein's early ideas on relativity were not at all known to Picasso at the time of consolidating his new cubist conceptions. More generally, it was not before 1919, when in the wake of the famous Eddington eclipse expedition Einstein was catapulted into world-fame, that the popularizations of relativity theory captured public and artistic

[33] (Linda Dalrymple Henderson 1983; Henderson 2004; Henderson 2005). And see also Henderson's contribution to this collection.

[34] The most salient recent version of which appears in (Miller 2001).

imagination.REF Only then, ideas of space and time related to relativity did offer new metaphors and opened new avenues of expression that some prominent artists undertook to follow. But as Henderson's work shows, it was not relativity, but some central ideas stemming from classical physics in the late nineteenth century that underlie the ways in which science contributed to creating new artistic directions in the early modernist period. These ideas were related above all with the ether, but also with other concepts and theories that stressed the existence of a supra-sensible, invisible physical phenomena. The latter included the discovery of X-rays, radioactivity, the discourse around the fourth dimension (especially as popularized through the works of the British hyperspace philosopher Howard Hinton (1853-1907)), and the idea of the cosmic consciousness introduced by the Russian esoteric philosopher Pyotr Demianovich Ouspenskii (1878–1947).

Henderson's detailed historical research is a superb example of how, by looking into the development of science, one can gain new insights into the issue of modernism in art. The main thread of her account emerges from within the internal development in the arts, and focuses on some crucial historical crossroads where substantial questions about the most fundamental assumptions of art and of its language arose at the turn of the twentieth century. Faced with these pressing questions, certain artists started looking for new ideas and new directions of thought with the help of which they might come to terms with such questions. Henderson then separately focused on contemporary developments in science, developments that in themselves had nothing to do with modernism or with some kind of modernist *Zeitgeist*, and she showed how these developments afforded new concepts, a new imagery, and new perspectives that the artists could take as starting point for the new ways they were looking for in their own artistic quest. Thus, in Henderson's narrative there is no assumption of common ideas or common trends simultaneously arising in both realms. In fact, whether or not the main scientific ideas were properly understood by the artists in question is not a truly relevant point in her account. She shows in this way, how scientific ideas—not necessarily the truly important or more revolutionary ones at the time—played an important role in the consolidation of central trends and personal styles in modernist art (Cubism, Futurism Duchamp, Boccioni, Kupka, etc.). Science appears here as offering a broadened world of ideas, metaphors, and images from which the artists could pick according to their needs, tastes and inclinations.

Henderson's works thus provides the kind of standards and the basic kind of approach that one would like to see in place (albeit by turning around the direction of the impact), in any serious attempt to making sense of modern mathematics as part of the more general cultural phenomena of modernism. This would involve an exploration of how internal developments in the discipline of mathematics led to critical crossroads that called for the kind of significant changes that we know to have affected mathematics in this period. Then, it would require showing how external input, coming from the arts, music, architecture or philosophy, could be instrumental in shaping the course of the ways taken by those mathematicians who led the discipline into the new directions that arose at the turn of the twentieth century. As already stressed above, although one should not rule out offhand the possibility of such kind of external impact having indeed taken place and having been meaningful in the history of mathematics, little evidence of anything of the sort has been put forward by historians (except, perhaps, to a very limited extent, in the case of philosophy).[35]

Moving to the opposite perspective, I would like to consider now the two more salient examples of analyses of modern mathematics as part of the more general cultural phenomenon of modernism, namely, the books of Herbert Mehrtens and, more recently, of Jeremy Gray. Mehrtens' *Moderne-Sprache-Mathematik* (Mehrtens 1990) was the first serious attempt to present an elaborate analysis of the phenomenon of modernism in mathematics, in which not only the internal history of the mathematical ideas had a prominent role, but also semiotic concepts and philosophical insights drawn from authors like Foucault or Lacan were significantly brought to bear. Indeed, Mehrtens' analysis accords prime importance to an examination of mathematical language, while stressing a three-fold distinction between different aspects of the latter: (1) mathematics *as* language

[35] An alternative, but not very convincing, way to connect mathematics with the general phenomenon of modernism appears in (Everdell 1998), where Cantor and Dedekind are presented as the true (unaware) initiators of modernism because the way in which they treated the continuum in their mathematical work.

(*Sprache Mathematik*), (2) the language used in mathematical texts (comprising systems of terms and symbols that are combined according to formal rules stipulated in advance), and (3) the language of mathematicians (*Sprechen der Mathematiker*), which comprises a combination of language used in mathematical texts, of fully formalized and of texts written in natural language (Mehrtens 1990, pp.402–525).

In these terms, Mehrtens discussed modernism in mathematics by referring to the main kinds reactions elicited by the development of mathematics by the end of the nineteenth century, particularly as they manifested themselves in debates about the source of meaning in mathematics and about the autonomy of the discipline. These reactions he described in terms of two groups of mathematicians espousing diverging views. On the one hand, there was a "modern" camp, represented by the likes of Georg Cantor (1845-1914), David Hilbert (1862-1943), Felix Hausdorff (1868-1942) and Ernst Zermelo (1871-1953). Characteristic of their attitude was an increasing estrangement from the classical conception of mathematics as an attempt to explore some naturally or transcendentally given mathematical entities (such as numbers, geometrical spaces, or functions). Instead, they stressed the autonomy of the discipline and of its discourse. They conceived the essence of mathematics to be the analysis of a man-made symbolic language and the exploration of the logical possibilities spanned by the application of the rules that control this language. Mathematics, in this view, was a free, creative enterprise constrained only by fruitfulness and internal coherence. The leading figure of this camp was, for Mehrtens, Hilbert.

Concurrently, a second camp developed, which Mehrtens denoted as "countermodern", led by mathematicians such as Felix Klein (1849-1925), Henri Poincaré (1854-1912), and Luitzen J. E. Brouwer (1881-1966). For them, the investigation of the spatial and arithmetic intuition (in the classical sense of *Anschauung*) continued to be the basic thrust of mathematics. The counter-modernists acknowledged the increasing autonomy of mathematics and its detachment from physical or transcendental reality, but they attempted to establish its certainty in terms of extra-linguistic factors, giving primacy to individual human intellect. The rhetoric of "freedom" of ideas as the basis of mathematics, initiated by Richard Dedekind (1831-1916)[36] and enthusiastically followed by the modernist mathematicians, was rejected by the mathematicians of the countermodernist camp, who priced above all finitness, *Anschaaung* and "construction". In Mehrtens' account, Brouwer appears as the arch-counter-modernist. His idiosyncratic positions in both mathematical and political matters (as well as the affinities between Brouwer and the national-socialist Berlin mathematician Ludwig Bierberbach (1886-1982)) allowed Mehrtens to identify what he saw as the common, counter-modernist traits underlying both levels.[37]

An important and original point underlying Mehrtens' analysis is the stress on the simultaneous existence of these two camps and the focus on the ongoing critical dialogue between them as a main feature of the history of early twentieth-century mathematics. This critical dialogue was, among other things, at the root of a crisis of meaning that affected the discipline in the 1920s (the so-called "foundational crisis" (pp. 289-330)) and led to a redefinition of its self-identity. Moreover, by contrasting the attitudes of the two camps, Mehrtens implicitly presented the modernist attitude in mathematics *as a matter of choice*, rather than one of necessity.

Mehrtens' book has been consistently praised for its pioneering position in the debate on modernism in mathematics, and for the original approach it has put forward. However, its limitations have also been pointed out. Mehrtens' analysis focuses mainly on programmatic declarations of those mathematicians he discusses and on their institutional activities. These are matters of real interest as sources of historical analysis and it is worth stressing that the contents of mathematics are influenced also by ideological considerations and institutional constraints. But as Moritz Epple has stressed, in the final account, "Mehrtens does not attempt to analyze some of the more advanced productions of modernist or counter-modernist mathematicians, and, in fact, he makes no claims about the internal construction of modern mathematics" (Epple 1997, p.191). Thus, Mehrtens leaves many fundamental questions unanswered and his discussion may be misleading. For one thing, the critical debate among "moderns" and "countermoderns"

[36] (Corry 2004, 64–76).

[37] See also (Mehrtens 1996).

would appear to be, in Merhtens' account, one that referred only to the external or meta-mathematical aspects, while being alien to questions of actual research programs, new mathematical results and techniques, or newly emerging disciplines. In addition, the classification of mathematicians into these two camps, and the criteria of belonging to either of them, seems too coarse to stand the test of close historical scrutiny. In this sense, Mehrtens' book, for all its virtues, falls short of giving a satisfactory account of modern mathematics as a modernist undertaking.

Having said that, I think that two basic elements of Mehrtens' analysis are highly relevant to any prospective, insightful analysis of modernism in mathematics. First is the possible, simultaneous existence of alternative approaches to mathematics that are open to choice according to considerations that do not strictly derive from the body of mathematics itself. Some of the elements that Mehrtens identifies in his distinction between modern and counter-modern seem to me highly relevant, but I think they could be more fruitfully used by historians if approached in a less schematic way, namely, by realizing that in the work of one of the same mathematician (or, alternatively, in the works of several mathematicians associated with one and the same school or tradition) we can find elements of both the modern and the counter-modern trend. These various elements may interact and continuously change their relative weight along the historical process. The second point refers to the historical processes that Mehrtens indicates as leading to the rise of modernist approaches in mathematics, namely the sheer rapid growth of the discipline (together with other branches of sciences) by the late nineteenth century, and the enormous diversity and heterogeneity that suddenly appeared at various levels of mathematical activity (technical, language-related, philosophical, institutional). In this sense Mehrtens follows the lead of those accounts of the rise of modernism in the arts that have presented it as a reaction to certain sociological and historical processes (such as urbanization, industrialization, or mechanization), and that in my view, if identified within the history of mathematics may lead us to gain some new insights on the development of the discipline.

The second book to be mentioned here is Jeremy Grays' more *Plato's Ghost. The Modernist Transformation of Mathematics* (J. J. Gray 2008). I will refer only briefly to it, as the reader may turn to Gray's article in this collection for more.[38] Gray's book provides a thoroughgoing account of the main transformations undergone by mathematics in the period that we are discussing here, and compares the main traits of these developments with the conceptions that previously dominated the discipline and that he schematically summarizes as "the consensus in 1880". His claim is that the developments so described are best understood as a "modernist transformation". This concerns not just the changes that affected the contents of the main mathematical branches, but also other aspects related to the discipline such as foundational conceptions, its language, or even the ways in which mathematics was popularized. Naturally, Gray is well-aware that "if the idea of mathematical modernism is to be worth entertaining it must be clear, it must be useful, and it must merit the analogy it implies with contemporary cultural modernisms".[39] In addition, "there should be mathematical developments that do not fit: at the very least those from earlier periods, and one might presume some contemporary ones as well". Accordingly, Gray's book opens with a characterization of modernism meant as the underlying thread of his analysis. Thus, in his own words:

> Here modernism is defined as an autonomous body of ideas, having little or no outward reference, placing considerable emphasis on formal aspects of the work and maintaining a complicated – indeed anxious – rather than a naïve relationship with the day-to-day world, which is the de facto view of a coherent group of people, such as a professional or discipline-based group that has a high sense of the seriousness and value of what it is trying to achieve. (J. J. Gray 2008, p.1)

Gray intends this definition not as a straight-jacket determined by a strict party line but rather as an idea of a broad cultural field providing a perspective that may help the historian integrate issues traditionally treated separately (including both technical aspects of certain sub-disciplines and prevailing philosophical conceptions about mathematics)[40], or stressing new historical insights on previously unnoticed developments. Thus, for instance, the interactions with ideas of artificial languages, the importance of certain

[38] See also (J. J. Gray 2006).

[39] REF to article here.

[40] See also (J. J. Gray 2004).

philosophers hitherto marginalized in the history of mathematics, the role of popularization, or the interest in the history of mathematics which had a resurgence in this period. One issue of particular interest raised by Gray in this context is that of "anxiety" (pp. 266-277). The development of mathematics in the nineteenth century is usually presented as a great success story, which certainly it is and Gray does not dispute it. But at the same time, a growing sense of anxiety about the reliability of mathematics, the nature of proof, or the pervasiveness of error, was a recurrent theme in many discussions about mathematics, and this is an aspect that has received much less attention. Gray raises the point in direct connection with the anxiety that is often associated with modernism as a general cultural trait of the turn of the century. Thus, he calls attention, as an example of this anxiety, to some texts in which such a concern is manifest and that historians previously overlooked or just regarded as isolated texts. Gray makes a clear and explicit connection of these texts, both among one another and with the broader topics of modernism.

Gray's book complements Mehrtens' in presenting a much broader and nuanced characterization of the discipline of mathematics in the period 1890-1930. On the other hand, in comparison with Mehrtens, Gray devotes much more attention to describing these characteristic features than to explaining the motivations and causes of the processes that ultimately led mathematics to become the kind of discipline that he aptly describes. I think that for the purposes of justifying the use of the of the idea of modernism in mathematics as a historiographical category with an truly added value may significantly benefit from a stronger focus on showing (if possible) that the processes that led to modernism in general and to modernism in mathematics are similar and have common cultural roots.

4. Greenberg's Modernist Painting and Modernist Mathematics

After this general overview of existing discussions of the recent historiography of mathematics, of modernism in general, and of the possible connections between art, science and mathematics at the turn of the twentieth century, I proceed to discuss in this and in the next section two specific kinds of analysis of modernism, completely unrelated with mathematics, but with the help of which I would like to explore the possibility of further broadening the scope of our discussion here. In the first place, I would like to discuss some ideas found in the writings of the celebrated and highly controversial art critic Clement Greenberg (1909-1994). I am aware, of course, that bringing the name of Greenberg as part of any discussion on modernism is a tricky matter. Indeed, for some historians of art Greenberg is total anathema and the foremost example of how the history of modern art should *not* be written and understood. Art historian Caroline A. Jones, for instance, described his views on modernism as “extraordinarily narrow” and as not proving “capacious enough for much painting of the modern period (even much “great painting”, *pace* Greenberg)”.[41] More recently, she published the most comprehensive account to date of Greenbergs’s writings and influence (Jones 2005), and the reader willing to take the challenge of her ambitious book will get the direct taste of the kind of passionate opposition (and attraction) that the “Greenberg effect” has aroused (and as the book instantiates – continues to arise) among its critics. But my point in calling attention to some of his texts is that I find them highly suggestive for the main aim of this article. Being an outsider to the world of art criticism, I can bypass the question of whether or not his characterization of modernism in art is comprehensive enough. Likewise, I can certainly ignore the ways in which he allegedly turned his view from *descriptive* to *normative*, i.e., that he did not limit himself to providing a historical explanation of the process that led to the creation and predominance of certain styles in twentieth century art, but he also wanted to determine, along the same train of ideas, what good art is and should be.[42] Greenberg was certainly not just a detached commentator but a main figure, strongly involved in the art scene in New York who had the power and the tools to build and destroy at will the careers of many an artist. His support of Jackson Pollock is a well-know chapter of his achievements in this regard, and so is his very negative attitude towards Marcel Duchamp and Ad Reinhardt.

[41] (Jones 2000, p.494).

[42] One can find in Greenberg’s text support got such a view, but in other places he emphatically denied that his analysis was ever intended as anything beyond pure description. See, e.g., (Greenberg 1983): “I wrote a piece called ‘Modernist Painting’ that got taken as a program when it was only a description.”

Good examples of the kind of insights that I deem truly valuable in Greenberg's texts can be found in a famous article of 1960, "Modernist Painting", where he characterized the essence of modernism in terms that, if unaware of the context, one could easily take to be a description of modern mathematics. He thus wrote:

> The essence of Modernism lies, as I see it, in the use of characteristic methods of a discipline to criticize the discipline itself, not in order to subvert it but in order to entrench it more firmly in its area of competence. … The self-criticism of Modernism grows out of, but is not the same thing as, the criticism of the Enlightenment. The Enlightenment criticized from the outside, the way criticism in its accepted sense does; Modernism criticizes from the inside, through the procedures themselves of that which is being criticized. (Greenberg 1995, p.85)

Indeed, the reflexive character of mathematics (discussed above) reached a distinctive peek at the turn of the twentieth century, and became the main tool for discussing and indeed criticizing the discipline. Think of the foundational works of Frege, Russell, Hilbert, Brouwer, Weyl or Gödel. As in Greenberg's description, this "criticism" worked from within, using the tools of the discipline, and it was meant not to subvert it, but rather to entrench its status.

For Greenberg, the source of this new kind of criticism coming from within could be traced back to Kant. It would seem natural that, given the essentially critical nature of the discipline, philosophy would engage in this kind to self-criticism, and Kant took it to new heights in his critical philosophy which explored the conditions of production of philosophy itself. But here Greenberg raised an interesting historical point which is relevant to our account: as the 18th century wore on, more rational justifications started to emerge in other disciplines as well, eventually reaching the arts. The latter, according to Greenberg, had been denied by the Enlightenment a serious task and the arts were thus gradually being assimilated to entertainment "pure and simple". A type of Kantian self-criticism that would explore, from within art itself, the conditions of production of art (and here he meant mainly the visual arts) appeared as a possible way to redefine the kind of experience that would stress what is valuable in art in its own right and, particularly, what could not be obtained from any other kind of activity. Herein lays Greenberg's explanation of the origin, the essence, and indeed the justification of modernist art:

> Each art had to determine, through its own operations and works, the effects exclusive to itself. … It quickly emerged that the unique and proper area of competence of each art coincided with all that was unique in the nature of its medium. (Greenberg 1995, p.86)

And in the case of painting this led Greenberg to characterize modernism in terms of a preoccupation with two main dimensions of this artistic activity, namely, (1) the intrinsic fact of painting's *flatness* and the inherent physical delimitation of this flatness, and (2) the gradual tendency of painting (recognizable since the last third of the nineteenth century) to estrange itself from the classical task of representation while occupying itself increasingly with questions pertaining to its own nature. Thus, these two main characteristic features, painting's preoccupation with the question of flatness and its limitations, appear here as a direct consequence of the self-critical processes that Greenberg described above:

> It was the stressing of the ineluctable flatness of the surface that remained, however, more fundamental than anything else to the processes by which pictorial art criticized and defined itself under Modernism. For flatness alone was unique and exclusive to pictorial art. The enclosing shape of the picture was a limiting condition, or norm, that was shared with the art of the theater; color was a norm and a means shared not only with the theater, but also with sculpture. Because flatness was the only condition painting shared with no other art, Modernist painting oriented itself to flatness as it did to nothing else. (Greenberg 1995, p.86)

Greenberg's focusing exclusively on the question of flatness *as the* defining feature of modernist art has been one of the main points of criticism directed against him. We need not enter a debate about that here. What I do learn from Greenberg's analysis, however, is a possible underlying explanation of the historical conditions for the rise to pre-eminence of what Greenberg sees as Kantian-like self-criticism (art analyzing art with the tools of art alone) and which appears as a main characteristic trait of modernist art. Since as already indicated, this kind of critical approach is also strongly characteristic of modern mathematics (and especially of the foundational quests typical of the turn of the twentieth century: mathematics analyzing the foundations and the limitations of mathematics with the tools of mathematics alone, and without the help of external, philosophical and metaphysical arguments) we are led to wonder about a possible new focal point of analysis that arises from Greenberg's approach to the question: was the rise of a new kind of modern mathematics related to a search for what was unique and exclusive to mathematics and to the nature of its medium? And if so: why did mathematics (mathematicians) engage in this search? What happened in, say, the last part of the nineteenth century, and not before that, that prompted this kind of search and what were the consequences of it? And we may then ask these questions for mathematics in general, and not just for those places where modernist trends have been mainly pursued, namely the new kind of foundational research that appeared in the works of Frege, Russell, Hilbert and others at the turn of the twentieth century. I will return briefly to these questions in the concluding section. At this point I just want to stress that the analogy with Greenberg's analysis would be meant to help us understand the origins and causes of the processes (social, institutional, disciplinary, philosophical, internal, etc.) behind the rise of modern mathematics and not just to check against a list of features characteristic of modernism in art.

It is enlightening to consider some additional points raised by Greenberg and that are relevant to our discussion. Thus, for instance, strongly connected with the previous issue, Greenberg stressed the centrality of the quest for autonomy of art. The impact of the process of self-criticism was translated, in Greenberg's analysis, to a focused search for "purity" in art as the guarantee for the preservation of the necessary standards,[43] and as a consequence, the status of the medium of art was transformed. In Greenberg's words:

> Realistic, naturalistic art had dissembled the medium, using art to conceal art; Modernism used art to call attention to art. The limitations that constitute the medium of painting -- the flat surface, the shape of the support, the properties of the pigment -- were treated by the Old Masters as negative factors that could be acknowledged only implicitly or indirectly. Under Modernism these same limitations came to be regarded as positive factors, and were acknowledged openly. Manet's became the first Modernist pictures by virtue of the frankness with which they declared the flat surfaces on which they were painted. The Impressionists, in Manet's wake, abjured underpainting and glazes, to leave the eye under no doubt as to the fact that the colors they used were made of paint that came from tubes or pots. Cézanne sacrificed verisimilitude, or correctness, in order to fit his drawing and design more explicitly to the rectangular shape of the canvas. (Greenberg 1995, p.86)

Again the analogy with mathematics seems to me highly suggestive, but we need to analyze its validity very carefully. The search for autonomy, and eventually even segregation, is an acknowledged characteristic of at least certain important parts of modern mathematics. In this sense the analogy with modern art is evident and has been

[43] A discussion of "purity" and its centrality in modernism, from a different perspective appears in (Cheetham 1991).

often mentioned. But, what were the reasons for this? We are well aware of important internal, purely mathematical dynamics of ideas leading to the rise of a new kind of approach and practice that stressed the need for the autonomy of mathematical discourse and mathematical methods. Here perhaps with the help of a perspective similar to that suggested by Greenberg for art, we may look for some additional, more external kinds of causes in the case of mathematics. It is clear that the increased search for purity in mathematics can be related to a specific attempt to "guarantee its standards of quality". But what about "limitations that constitute the medium" of mathematics, that were treated by the Old Masters as negative factors that could be acknowledged only implicitly or indirectly", and that in modern mathematics could come "to be regarded as positive factors" and to be "acknowledged openly". This appears as a remarkable, and far from self-evident characterization of modern art that Greenberg's analysis brings to the fore. It does not seem, however, tha it can be transposed to mathematics in a straightforward manner. This has to do with the differences I mentioned above between mathematics and the arts, and the stronger role played in the former by continuity than in the latter. In further exploring this point, however, one might try to bring to bear ideas from sociologists of science such as Rudolf Stichweh, who has stressed the systemic, interrelated character of discipline formation by the end of nineteenth century. Stichweh's analysis shows how the emergence and consolidation of an autonomous self-understanding of the various academic disciplines depended always on similar process taking place in the neighbouring disciplines at the same time (Stichweh 1984). But at this point I leave this as an open question that calls for further thought that might lead to new insights on the question of modernism in mathematics.

In referring to the "necessity of formalism" as the "essential, defining side" of modernism (at least in the case of painting and sculpture), Greenberg added another interesting explanation that seems very suggestive for mathematics as well:

> Modernism defines itself in the long run not as a "movement", much less a program, but rather as a kind of bias or tropism: towards aesthetic value, aesthetic value as such and as an ultimate. … This more conscious, this almost exacerbated concern with aesthetic value emerges in the mid-nineteenth century in response to an emergency. The emergency is perceived in a growing relaxation of aesthetic standards at the top of Western society, and in the threat this offers to serious practice of art and literature. (Greenberg 1971, 171)

Keeping in mind that terms such as “formal”, “abstract” or “aesthetic” have significantly different meanings and elicit different contexts in mathematics and in the arts, one can still ask whether the idea of associating the entrenchment of formalist approaches as part of the consolidation of modern mathematics with the a reaction to an emergency, as described here by Greenberg for the arts, may bring with it new insights. Moreover, we can also ask if the “emergency” in question was not only similar, but perhaps even the same one in both cases. I already mentioned the issue of “anxiety” discussed by Gray in relation with the development of mathematics at the turn of the nineteenth century, and which he related to what some mathematicians conceived as a relaxation of standards. There is no doubt that formalism in mathematics can be related to a possible reaction of such a relaxation. Thus, formalism may appear here not just as a common trait that can be perceived in both mathematics and art, but also as motivated by similar concerns in both cases. More on this I will say in the next section.

Finally, I would like to mention yet another suggestion of Greenberg that may be relevant for historians of mathematics in their own field, as it touches upon the question of the supposed radical break with the past that appears in so many characterizations of modernism. In an article entitled “Modern and Postmodern”, Greenberg wrote:

> Contrary to the common notion, Modernism or the avant-garde didn't make its entrance by breaking with the past. Far from it. Nor did it have such a thing as a program, nor has it really ever had one. It's been in the nature, rather, of an attitude and an orientation: an attitude and orientation to standards and levels: standards and levels of aesthetic quality in the first and also the last place. …
>
> And where did the Modernists get their standards and levels from? From the past, that is, the best of the past. But not so much from particular models in the past -- though from these too -- as from a generalized feeling and apprehending, a kind of distilling and extracting of aesthetic quality as shown by the best of the past. (Greenberg 1980)

I find it remarkable that Greenberg would stress this point in opposition to what so many considered an unavoidable trait of modernism, since as I said above, real radical breaks with the past seem rather unlikely in mathematics. As Greenberg stresses here, modernism may arise not from a radical break, but rather from a conscious process of distilling and extracting quality from what proved to be the best practice in the past. I think that in laying the central elements of modern mathematics, some of the most influential mathematicians of the turn of the century acted precisely in this way. This was certainly the case, as I have discussed in detail elsewhere, with Dedekind's early introduction of structuralist concerns in algebra[44] and with Hilbert's introduction of the modern axiomatic approach.[45]

As already stated, however, there are also good reasons to react to Greenberg's views with great care. It is not only that they are very much debated among historians of arts, it is also that Greenberg did not write systematic, scholarly texts with all due footnotes etc. Most of his writings appeared as scattered articles, conferences, etc. and they sometimes follow a somewhat associative style. Thus, one must not be surprised to find deep changes and possibly conflicting views in them throughout the years.

And yet, even if the criticism directed at him is well taken, especially when one tries to apply his view to this or that artist in particular while closely examining the details, this does not mean that the essential structure of the processes he describes cannot be reconstructed for the purposes I am pursuing here, and then followed in a more scholarly solid fashion. If one is able to develop explanations of these kinds for mathematics, then it may turn out that it is not only justified and useful to use the term modernism in the context of the history of mathematics, but also that it is not just a coincidence that modernism appears in mathematics as well as in the arts nearly contemporarily, and that this coincidence can be made sense of in more or less tangible terms.[46]

[44] (Corry 2004c, chap.2).

[45] (Corry 2004a).

[46] Greenberg, of course, is not the only one to discuss modernism in terms of the processes that led to its rise, rather than by just providing a checklist of characteristic features. I may also mention here the work of Dan Albright (Albright 1997; 1999), who stresses the crisis of values in art that led to modernism. In his view, if in previous centuries, artists, writers and musicians could be inherently confident about the validity of the delight and edification they provided to their audiences, during the twentieth century art found itself in a new and odd situation, plagued with insecurity. Faced with the crisis, radical claims about the locus of value in art were advanced in various realms at nearly the same time. The various radical modernist manifestoes thus produced reflect the need of the artist not only to create, as was always the case in the past, but also to promote new standards of value and to provide some new kind of justification to the very existence of art.

5. Wittgenstein's Vienna and Modernist Mathematics

A second source where I would like to look for ideas relevant to a possible discussion of modernism in mathematics is the book *Wittgenstein's Vienna* by Allan Janik and Stephen Toulmin. The main topic of this book is an interpretation of Wittgenstein's *Tractatus* (a quite radical re-interpretation at the time of its publication) that refutes the accepted views according to which the basic questions that underlie this seminal and highly influential work were epistemology, philosophy of science, and logic taken for their own sake. Rather, the authors aimed at presenting Wittgenstein as a thinker deeply rooted in the intellectual life of Vienna at the turn of the twentieth century, for whom the question of language and its limitations was mainly an *ethical* concern and not merely a linguistic-analytic one. These ethical concerns, the authors contended, can only be fully understood against the background of Viennese modernism in its various manifestations. Thus, they opened their discussion by posing the following, basic question that is pursued thorough the book:

> Was it an absolute coincidence that the beginnings of twelve-tone music, 'modern' architecture, legal and logical positivism, nonrepresentational painting and psychoanalysis – not to mention the revival of interest in Schopenhauer and Kierkegaard – were all taking place simultaneously and were largely concentrated in Vienna? (Janik & Toulmin 1973, p.18)

The central hypothesis of the book is that "in order to be a fin-de-siècle Viennese artist or intellectual conscious of the social realities of Kakania [a term coined by Robert Musil to disparagingly describe Austro-Hungarian society (L.C.)] one had to face the problem of the nature and limits of language, expression and communication. (p. 117)" Accordingly, they offered an account of the deep changes that affected art, philosophy, and various other aspects of cultural life around 1900 in Vienna, as interrelated attempts to meet the challenges posed by questions of communication (language), authenticity, and symbolic expression. And the most important instance of the philosophical side of this sweeping cultural phenomenon they found in the work of the man whose writings, in their view, embodied the crucial influence on Wittgenstein, namely, the somewhat forgotten author Fritz Mauthner (1849-1923).

Mauthner, who developed a unique doctrine of "Critique of Language" (*Sprachkritik*) in several interesting books, is one of the few persons mentioned by name in the *Tractatus*.

In the received interpretation of Wittgenstein, the importance of this reference to Mauthner is often downplayed, but Janik and Toulmin make it the centerpiece of their analysis. I am not particularly interested here in adjudicating these conflicting views about Wittgenstein and his work. But the alternative (and in my view enlightening) approach offered by Janik and Toulmin certainly affords a useful perspective for our discussion here, since they did not limit themselves to indicating general analogies between various fields of activity, or a common, underlying *Zeitgeist*, but rather emphasized concrete historical processes that were motivated by similar concerns stemming from the specific historical circumstances of turn-of-the-century Vienna.

Incidentally, an important focus of attention for Janik and Toulmin is found in contemporary science, and in particular in the works of Ernst Mach (1838-1916), Heinrich Hertz (1857-1894) and Ludwig Boltzmann (1844-1906). For these three scientists, as it is well know, metaphysics had no place in science, and they devoted conscious and systematic efforts at finding those places where metaphysics had subtly but mistakenly been incorporated. This task, however, was not pursued in the same way by the three of them. Janik and Toulmin describe them as representing significantly different stages in a continuous process. Mach represents a first stage where the limits of physics were set "externally", as it were, by means of a more philosophical kind of analysis. Hertz and Boltzmann, on the contrary, by following an approach that can retrospectively be described as "axiomatic", pursued the same task "from within". Hertz and Boltzmann sought to set the correct limits of physical science by means of an introspective kind of analysis, using the tools of science (and here, of course, we find a remarkable similarity with Greenberg's analysis as explained above).

The interesting point in their analysis, however, is that Janik and Toulmin embedded this two-stage process in the more general, broad historical processes that underlie all other manifestations of Viennese modernism. And first and foremost among these manifestations was, for them, the one pertaining to the processes leading from Mauthner to Wittgenstein. The philosophical critique of language undertaken by Mauthner as a response to the above mentioned need to establish the "limits of language, expression and communication" starts from a point which is similar to that of Mach's attempt in physics.

And very much like Hertz and Boltzmann continued with Mach's quest, but by way of an alternative, more internally focused path, so did Wittgenstein in relation to Mauthner. Hertz and Boltzmann, according to Janik and Toulmin, "had shown how the logical articulation and empirical application of systematic theories in physical science actually gives one a direct *bildliche Darstellung* of the world … namely, a mathematical model which, when suitably applied, can yield true and certain knowledge of the world. And they had done so, furthermore, in a way that satisfied Kant's fundamental antimetaphysical demands – namely, by mapping the limits of the language of physical theory entirely 'from within'." (p.166) In similar terms, Janik and Toulmin presented the philosophical work of Wittgenstein as a continuation of Mauthner's, in which also the limits of language in general were mapped from within. They also examined and lay all the necessary stress on the ethical outlook which in their interpretation was so central to Wittgenstein's undertaking and that arose from the writings of Kierkegaard and Tolstoy. (Of course, this central element played no role in the story told about Mach, Hertz and Boltzmann.)

The socio-cultural elements underlying both aspects of the story as described above are expanded in a subtle way – in the account of Janik and Toulmin – to cover other fields of activity, along the same lines: music, architecture, journalism, law, painting and literature. And in all of these fields they also added a third stage that was produced along the lines of a commonly characterized historical processes. Thus, for instance, the three stages in music are represented by Gustav Mahler, then Arnold Schönberg, then Paul Hindemith. In the case of architecture it is OttoWagner, then Adolf Loos and then Bauhaus.[47] And in the case of philosophy, the stage after Wittgenstein (who came after Mauthner) is that of logical positivism. The process which is common to all these threads can be briefly described as follows:

> In architecture as in music, then, the technical innovations worked out before 1914 by the 'critical' generation of Schönberg and Loos were formalized in the 1920s and 1930s, so becoming the basis for a compulsory antidecorative style which eventually became as conventional as the overdecorative style which it displaced. And we might pursue these parallels still further if we pleased – into poetry and literature, painting and sculpture, and even

[47] An analysis that complements this view and locates the Bauhaus movement in relation with logical positivism, as part of Viennese modernism is found in (Galison 1990).

> into physics and pure mathematics. In each case, novel techniques of axiomatization or prung rhythm, operationalism or nonrepresentational art, were first introduced in order to deal with artistic or intellectual problems left over from the late nineteenth century – so having the status of interesting and legitimate new *means* – only to acquire after a few years the status of *ends*, through becoming the stock in trade of a newly professionalized school of modern poets, abstract artists or philosophical analysts. (p. 254)

Not *Zeitgeist*-like arguments or superficial analogies, then, as part of the explanation, but a common ground to all these processes, namely, "a consistent attempt to evade the social and political problems of Austria by the debasement of language."

Can the insights of Janik and Toulmin be imported into the history of mathematics in a fruitful way? It is curious that in the passage just quoted they mentioned pure mathematics as having been affected by the same circumstances as other cultural manifestations. They do not give any details about what they may have had in mind when saying this, and indeed at a different place they did state that "in a very few self-contained theoretical disciplines – for example, the purest parts of mathematics – one can perhaps detach concepts and arguments from the historico-cultural milieus in which they were introduced and used, and consider their merits or defects in isolation from that milieu." (p. 27) But the point is not what actually was done in the book in relation with mathematics, but what could be done by analyzing Viennese mathematics at the turn of the century from the perspective put forward by the book.

Given the programmatic style of this article I will not attempt to answer such questions in full here. But I do want to indicate certain parameters that could perhaps be taken into consideration in an attempted answer. In the relevant period of time, Vienna did have an interesting, original, and very productive mathematical community. Its more prominent names included Wilhelm Wirtinger (1865-1945), Philipp Furtwängler (1869-1940), Eduard Helly (1884-1943), Kurt Gödel (1906-1978), Kurt Reidemeister (1893-1971), Witold Hurewicz (1904-1956), Walther Mayer (1887-1948), Johann Radon (1887-1946), Alfred Tauber (1866-1942), Olga Taussky (1906-1995), Heinrich Tietze (1880-1964) and Leopold Vietoris (1891-2002!!). Each of these mathematicians arrived in Vienna from different places at different times, bringing their own baggage drawn from the mathematical traditions from which they stemmed. In addition, one cannot fail to notice that some of the most prominent members of the literary milieu in Vienna had a formal training in mathematics. The three most prominent examples of this are Robert Musil (1880-1942), Herman Broch (1886-1951) and Leo Perutz (1882-1957). The latter even continued to be actively involved in mathematics throughout his life.[48] Of course, even before we start to consider the question that occupies us here, one should be able to come forward with a more articulate understanding of the Vienna mathematical community than we now have: what were the main mathematical fields pursued, what kinds of interactions existed with the local scientific communities and with neighboring mathematical institutions, what were the internal mechanisms of production, training and transmission of mathematical knowledge, etc. As already said, such questions have been pursued in varying degrees of detail and from different perspectives for Göttingen and Berlin, for the various USA centers of mathematics, and for some British and Italian contexts, but much less so for Vienna.

One piece of historical research that has been done for the mathematics at Vienna does indicate, however, that there might be some room for further pursuit of the question in the broader terms of modernism, as suggested above. Indeed Moritz Epple has written a historical account of Kurt Reidemeister's work of the 1920s on Knot Theory, while comparing it with work conducted simultaneously on the same field by at Yale. Epple does touch upon the intellectual atmosphere of the city as part of the relevant intellectual background to Reidemeister, and indeed his discussion of the rise of modern topology is framed in the broader context of modernism in mathematics (Epple 1999, 299–322). Thus for instance we learn of Reidemeister's connections with Hans Hahn, Otto Neurath, Otto Schreier and Karl Menger, all of them engaged in the activities of the Vienna Circle.

What kind of mathematics was done at the time in Vienna? To what extent such kind of mathematics can be proper called modernist? Is this somehow connected with the work and the person of their Viennese neighbor Boltzmann? As already suggested one might try to pursue these kinds of questions in detail. The focal point of the questions, according to the approach suggested here would be if this kind of mathematics is peculiar and different to what preceded it, and, more importantly, if the processes leading to the

[48] (Sigmund 1999).

changes that brought abut this possibly new conception are similar or similarly motivated as all the other complex processes described in Janik and Toulmin's book.

It is relevant to stress that Epple's main tool for the comparison he undertook in this case is based on the idea of "epistemic thing", originally introduced by historians of experimental sciences.[49] Epple uses the idea in order to explain in what senses Reidemeister's topological research differed from other contemporary topological, or mathematical research. Epple indeed suggests the plausibility that the specificity of this type of mathematical work was tied to certain facets of the intellectual milieu, even though, the nature of the tie remains to be explained.[50] In other words: Epple's work suggests that the Viennese intellectual modernism may help understand why Reidemeister was indeed developing the specific kind of mathematics that he did. In an ideal study of the mutual relationship between modernism and mathematics one might also be led to go the opposite direction, namely by understanding the specifics of Reidemeister's, and Hahn's, and Menger's, one may be led to uncover new historical mechanisms behind the development of Viennese modernism.

A different, and perhaps highly relevant direction in which the analysis of Janik and Toulmin can be illuminatingly taken by historians of mathematics has to do with the development of the modern axiomatic approach. I have devoted considerable attention in my own research to the work of David Hilbert, to the centrality of the axiomatic approach for his work and to the great impact that this aspect of his work had on mathematics and physics in the early twentieth century, precisely at the time under discussion here. In my analysis I have shown how the work of Hertz and Boltzmann had a direct influence on Hilbert and on the consolidation of the axiomatic approach, and its application to both geometry and physics. I have also stressed the pervasive presence of Mach's ideas and of his empiricist-oriented criticism in the background of all of Hilbert's work.[51] Now, it is remarkable that in the three-stage model of Janik and Toulmin precisely this thread, leading from Mach to Hertz and Boltzmann, which the authors single out as so highly

[49] (Rheinberger 1997).

[50] As Epple indicates, an important addition to explaining the intellectual background to *fin-de-siècle* Vienna is (Schorske 1981).

[51] (Corry 2004a, chap.2–3).

important, is not completed with its third stage. My suggestion here is that one might look at the process leading to, and at the consolidation of, Hilbert's new axiomatic approach precisely as that third stage. A historical analysis of the kind I have provided for Hilbert may plausibly be complemented with an eye on the kinds of processes described by Janik and Toulmin. In this way, the mathematics embodied in and promoted by Hilbert's approach could be seen as an aspect of mathematical modernism, not just because of a series of characteristic features associated with it, but rather because it might be seen as the outcome of a process with specific historical-cultural roots that gave rise to modernism in so many fields of culture at the time. This could be an excellent test, to be left for the future, of the possible usefulness of the term modernism in understanding the history of twentieth century mathematics as explained here.

6. Summary and Concluding Remarks

An emphasis on the formal, as opposed to thematic values; a rite of passage through avant-garde; a radical break with tradition (or even a "desire to offend tradition"); the wish to explore subjective experience as opposed to representing "outward experience"; a high degree of self-consciousness; a criticism of the basic principles of the discipline and of its limits using the tools of the discipline. These are some of the characteristic features typically associated with modernism in its various cultural manifestations at the turn of the twentieth century. Some of them were mentioned and analyzed by the authors whose works we examined above (works intended as an illustrative, rather than exhaustive, sample of scholarly discussions on the topic of modernism). Some of them are mentioned elsewhere as well, and it is clear, at any rate, that we may find such basic attitudes also in the mathematics of the period in question. Thus, historians of modern mathematics might debate the degree to which they are central and pervasive, and hence the extent to which it may be appropriate to describe modern mathematics as a modernist endeavor. My proposal in this survey is, however, that rather than just trying to explore the topic in this straightforward way, we should ask ourselves if the perspective of modernism may lead us to new insights in making sense of the history of modern mathematics. I hope that my analysis above may be taken to suggest some ways in which this could be done.

Take for instance Jeremy Gray's emphasis on the sense of anxiety that arose at the end of the nineteenth century side-by-side with the enormous successes of the discipline. Talk about this success is standard in any historical account concerning this period, but the concomitant anxiety indicated by Gray has much less been discussed (if at all). By situating it in a modernist context, Gray draws our attention to the possibility that this is a more significant issue than he have realized thus far. He gives the example of an inaugural lecture delivered in 1910 at Tübingen by Oskar Perron (1880-1975). Perron was a proficient mathematician with acknowledged contributions to various fields, but he was not in the same line of prominence as Hilbert or Noether. Thus, one will not find his name often mentioned in general discussions about mathematical modernism. But as Gray indicates, it is important to hear what a mathematician like him had to say about his discipline at the turn of the twentieth century. And in his lecture, Perron addressed mainly questions related to the gap between the public perception of mathematics and the real practices in the discipline, and particularly in relation with the question of the certainty and exactness of its methods.[52] What seems really interesting to me in this regard is the question whether this is an isolated phenomenon or a manifestation of a more generalized concern of the practitioners of mathematics at the time.

Well, if we follow the lead opened here by Gray, we do find instances that give us further food for thought. Thus, for instance, an interesting text of Alfred Pringsheim (1850-1941), who, like Perron, was a well-known mathematician, though not in the league of those usually discussed in relation with modernism. In addition to his mathematical activity, Pringsheim was deeply immersed in the broad cultural trends of his time, and that to an unusual degree. He came from a very wealthy Jewish family in Berlin who used its wealth to support art, and Alfred became himself a well-known art collector. He had a strong, well-cultivated musical background and he became one of the earliest Wagner supporters. The family house in Berlin and his own one in Munich (both known as "Palais Pringsheim") were prominent architectural icons (though they were far from any clue of modernist taste).[53] In 1904, on the occasion of the 145th anniversary of the

[52] (Perron 1911).

[53] (Lederer 2004)

Munich Academy of Sciences, Pringsheim gave a lecture entitled "On the Value and Alleged Lack of Value of Mathematics".[54] Without going into the details of the talk, I will just say that it reflects very closely the same kinds of concerns as Perron's. And, incidentally, Pringsheim had been one of Perron's most influential teachers in Munich.[55]

One may mention additional texts that go in the same direction,[56] and, more importantly, one is motivated now to look for more. Still, the question remains open whether we can find not just additional texts that serve as evidence for these kinds of concerns (which is quite likely), but rather if we can understand the real roots of it, and the processes leading to its rise and consolidation. And more specifically: whether these roots may be found to be directly connected, or at least closely related, to those at the basis of modernism as a broad cultural phenomenon (which I think is less likely, though still plausible).

The latter question relates to some of the issues discussed above throughout the article and that pertain to the sources of modernism. Thus for instance, Greenberg's explanation of the rise of modern painting in the need of each art, by the late nineteenth century, to determine, purely with the help of its own means, what was unique and exclusive to itself, according to the nature of its medium. The intense foundational activity which is one of the acknowledged characteristic features of mathematics at the turn of the century can be easily seen as a similar manifestation – within the discipline – of the phenomenon indicated by Greenberg in the case of painting. Indeed, this is a point typically stressed in the debates about modernism in mathematics. But, having said that, can we in addition explain the timing and the main thrusts of this foundational activity on the same grounds that Greenberg adduced for the arts? For Greenberg, for instance, on the wake of the Enlightenment the arts were gradually being assimilated to entertainment pure and simple, and this was a main trigger that led to the kind of internally pursued self-criticism laying at the basis of modernism. Can we come up with a similar explanation in the case of mathematics? For Dan Albright, for instance, the roots of modernism are related to the fact that art found itself in a new and odd situation, plagued with insecurity, opposed to

[54] (Pringsheim 1904).

[55] (Frank 1982)

[56] See, e.g., (von Mises 1922). And of course one can think in this context of all the texts related to the so-called foundational crisis of the 1920s.

the confidence on the validity of the delight and edification it had provided to their audiences in previous times. Perhaps this could be a fruitful lead to follow, in connection with the topic of anxiety just mentioned above. Can we trace a direct connection between the changes of status in the arts, with related changes of status in mathematics, with questions about certainty and the unity of mathematics, and with the increased trend of foundational research at the turn of the twentieth century? Can the explanations of Janik and Toulmin about the centrality of problem of language in the modernist culture of Vienna be of any help in trying to consolidate such an explanation?

Answering these questions would require, I believe, additional historical research that might turn up to be illuminating, though there is no warranty that new, interesting insights will arise up from it. Such an historical research would need to take into consideration that modernism is a historical phenomenon with an internal evolution and geographical specificities that are often overlooked. Thus, with the Great War in Europe precisely in the middle of the period that frames our discussion (1890-1930) and its profound social and cultural impact it is obvious that a single idea of “modernism” is too coarse to account for all the developments typically related with the term without further historicizing it. What is less obvious, but no less significant, are the differences among modernist cultures across the continent and in the USA. As mathematics is the quintessential universal endeavor, these geographical differences would seem irrelevant for the discussion, but I want to contend that they are not, and that the right way to consider modernism in mathematics, would be, if at all, at the local level: modernist Paris mathematics, modernist Viennese mathematics, etc. Moreover, this approach would inherently emphasize the need to analyze not just the pronouncements of the Hilberts and the Weyls but also the pronouncements and the mathematical deeds of the Pringsheims the Perrons and the Rademachers.

How useful is, then, the term ‘modernism’ for understanding the history of early twentieth-century mathematics? I hope to have shown that while the answer to this question may potentially be positive, there is a long way to go before this potentiality can be realized. In particular, a plain characterization of what modernism is (which is a much debated question anyway) will not suffice. What may be of use for gaining new insights into the history of mathematics from the perspective of this question a deeper understanding of the historical processes leading to modernism in its various cultural manifestations.

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