The scientific content of the letters is remarkably rich, touching on the difference between Borel and Lebesgue measures, Baire's classes of functions, the Borel–Lebesgue lemma, the Weierstrass approximation theorem, set theory and the axiom of choice, extensions of the Cauchy–Goursat theorem for complex functions, de Geöcze's work on surface area, the Stieltjes integral, invariance of dimension, the Dirichlet problem, and Borel's integration theory. The correspondence also discusses at length the genesis of Lebesgue's volumes *Leçons sur l'intégration et la recherche des fonctions primitives* (1904) and *Leçons sur les séries trigonométriques* (1906), published in Borel's *Collection de monographies sur la théorie des fonctions.*

Choquet's preface is a gem describing Lebesgue's personality, research style, mistakes, creativity, and priority quarrel with Borel. This invaluable addition to Bru and Dugac's original publication mitigates the regrets of not finding, in the present book, all 232 letters included in the original edition, and all the annotations (some of which have been shortened).

The book contains few illustrations, some of which are surprising: the front and second page of a catalog of the editor Gauthier–Villars (pp. 53–54), and the front and second page of Marie Curie's Ph.D. thesis (pp. 113–114)! Other images, including photographic portraits of Lebesgue and Borel, facsimiles of Lebesgue's letters, and various important academic buildings in Paris, are more appropriate. A portrait of Baire, whose name appears in half of the letters, could have been included as well. The index by names and index by topics are most useful, and the bibliography, taken from the original edition, is more than two hundred items rich.

This selected correspondence from Lebesgue to Borel is of interest to mathematicians and historians of mathematics. It provides invaluable information about the life and personality of Lebesgue, the mathematical atmosphere in France between 1900 and the First World War, and the genesis of Lebesgue's mathematical ideas.

Jean Mawhin
Département de mathématique, Université Catholique de Louvain,
B-1348 Louvain-la-Neuve, Belgium
E-mail address: mawhin@math.ucl.ac.be

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David Hilbert and the Axiomatization of Physics (1898–1918). From *Grundlagen der Geometrie* to *Grundlagen der Physik*

By Leo Corry. Archimedes. New Studies in the History and Philosophy of Science and Technology, vol. 10. Dordrecht/Boston/London (Kluwer Academic). 2004. ISBN 1-4020-2777-X. xviii + 513 pp. $179

The book is a comprehensive exposition of more than a decade of the author's research on Hilbert's involvement with physics. When Corry embarked on this research project back in the mid-1990s, only a few experts knew that Hilbert had worked in the field of theoretical physics and that the Mathematics Library at the University of Göttingen held a considerable number of *Ausarbeitungen* of lecture courses by Hilbert that dealt with physics. To be sure, Arnold Sommerfeld had already called for the publication of those very lecture notes in his eulogy for Hilbert, spoken at his grave in 1943, and indeed two volumes on physics will be included in the multivolume edition of *David Hilbert's Lectures on the Foundations of Mathematics and the Natural Sciences* [Hallett and Majer, 2004; further volumes in preparation]. In his biographical essay of 1944, Hermann Weyl had mentioned an involvement with physics, lasting from 1910 to 1922, as one of the five phases in his periodization of Hilbert's work. Physicists did, and in fact still do, refer to Hilbert spaces in quantum theory and to the Einstein–Hilbert action in general relativity. His involvement in the latter field had also been the subject of a few historical papers commenting on Hilbert's role in the genesis of Einstein's general theory of relativity. Yet, in the early 1990s, Hilbert was still seen as a mathematician par excellence, as the hero of formalistic mathematics, and as the father of a formalistic understanding of axiomatization that is at the core of today's foundations of mathematics.

It is, above all, this image of Hilbert that Corry set out to call into question by looking into Hilbert's research in physics, as documented by his published and unpublished work, and by taking into account the broader context of

Hilbert's work in the setting of his Göttingen locale. With the results of his investigations, some of which were published previously and have now been unified into an impressive comprehensive account, Corry strives to counteract and complement our received image of Hilbert as the logician and pure mathematician who kept aloof from applications. And this point is well taken. The book presents compelling evidence that Hilbert's interest in physics not only breaks the boundaries of Weyl's all-too-strict periodization, but pervades his whole intellectual biography.

The book is organized into nine chapters. In the first two, Corry takes us back into the 19th century, discusses Hilbert's early career, and sets the stage for his story by discussing the origins of Hilbert's foundational work in the pertinent mathematical and physical literature of the day. Specifically with respect to physics, Corry discusses at some length work by Carl Neumann, Heinrich Hertz, Paul Volkmann, Ludwig Boltzmann, and Aurel Voss. He shows that a foundational concern was shared by all of these writers in their critical expositions of classical mechanics on which Hilbert could draw when reflecting on the logical structure of this theory from his own point of view. Corry also goes into some detail on the origin of Hilbert's famous *Foundations of Geometry* as a context for the sixth of his famous 23 problems of 20th-century mathematics presented in 1900 to the International Congress of Mathematicians in Paris. Corry rightly takes this somewhat odd sixth problem, asking for an axiomatization of physics, to be a driving motive behind much of Hilbert's subsequent involvement with the natural sciences.

The third chapter of Corry's account then presents a discussion of a lecture course from 1905, entitled *Logical Principles of Mathematical Thought*. The lecture is of great significance for Corry's concerns, since in this course, Hilbert first attempts to exemplify his axiomatic analysis across various physical theories. Before going into details of Hilbert's lecture, Corry reminds the reader of the broader historical context of physics research at the time. As we know, Hilbert was operating in the midst of a circle of leading mathematicians and physicists who were concentrated in Göttingen as a result of Althoff's successful efforts to make Göttingen a center of physical and mathematical research in Prussia. Felix Klein, Hermann Minkowski, Carl Runge, Emil Wiechert, Max Abraham, and later Max Born, Emmy Noether, and many other mathematicians and physicists were in direct contact with Hilbert when he entertained his reflections on the axiomatic foundations of contemporary physics. Corry then provides the reader with a hands-on discussion of various axiomatizations that Hilbert gave in his 1905 lecture course. The axiomatic method, if we may call it that for now, was indeed applied by Hilbert to classical mechanics, thermodynamics, probability calculus, kinetic theory of gases, insurance mathematics, and electrodynamics, as well as psychophysics.

In the fourth chapter, Corry shifts the focus away a little from Hilbert and toward Minkowski, Hilbert's closest friend and colleague. Not only was Hilbert instrumental in getting Minkowski a call to Göttingen, but Corry also shows how Minkowski's approach to physics resonates with Hilbert's and how the two began to collaborate on the same agenda after Minkowski's arrival in Göttingen in 1902. Indeed, it was above all Minkowski's analysis and reformulation of special relativity and his elaboration of a Lorentz-covariant formulation of electrodynamics, published posthumously by Max Born, that would provide a central point of departure for all of Hilbert's subsequent work in this field. But it was only after Minkowski's sudden death in early 1909 that Hilbert really focused his attention almost exclusively on an attempt to come to an axiomatic understanding of physics at large. Corry discusses the first phase of this period, the years 1910–1914, in his fifth chapter as a transition on Hilbert's part from a mechanical toward an electromagnetic worldview or, as Corry calls it, reductionism. To the extent that one can pinpoint a commitment to either side of this pervasive debate at the beginning of the 20th century, Hilbert's lectures reflect the fact that Göttingen was a stronghold of the electromagnetic worldview at the time. This is witnessed by the central role that predominantly electromagnetic theories of the electron and its properties played for Hilbert's understanding of the structure of matter. Corry also discusses Hilbert's analysis of the Boltzmann equation by means of his theory of linear integral equations, as well as Hilbert's axiomatization of Kirchhoff's radiation laws. These episodes not only led to the first published papers by Hilbert in the field of physics proper; the excursion into the foundations of radiation theory also sparked a vivid controversy between Hilbert on the one hand, and Max Planck and Ernst Pringsheim on the other. The controversy focused most interestingly on the issue of whether and to what degree physically relevant facts are and need to be captured by mathematical axioms, thus reflecting the general problem of a proper demarcation between mathematics and physics.

In the next three chapters, which deal with the background of this image of Hilbert as a mathematician with a deep and genuine interest in physics, Corry tells the main part of his story: Hilbert's path to and involvement in the genesis of the general theory of relativity. Einstein's publication of the gravitational field equations of general relativity in late November 1915 and Hilbert's virtually simultaneous publication of essentially equivalent equations in terms of an action principle have been the subject of an intense debate among historians of science in recent years. This debate was fuelled by Corry's discovery of page proofs for Hilbert's communication to the Göttingen Academy of Sciences and the subsequent publication in 1997 of a widely read paper on the implications of this finding for the history of general relativity [Corry et al., 1997].

Corry begins to tell the story by an exposition of what he rightly calls the two pillars of Hilbert's unified theory of the foundations of physics: first, a generalized theory of Maxwellian electromagnetism advanced in 1912 by Gustav Mie; and second, Einstein's generalized theory of relativity and his theory of gravitation in the so-called "Outline" of this theory, coauthored with Marcel Grossmann in early 1913. Mie's theory amounted to a modification of Maxwell's equations, by suggesting that terms might be added to the usual Lagrangian which, on the carrying out of the variation, would produce modified field equations while preserving Lorentz covariance. These field equations, Mie hoped, would then agree with Maxwell's equations macroscopically, but might also allow for particle-like solutions in the microscopic realm. The appealing aspect of Mie's approach was that an interpretation of such field solutions as a representation of the fundamental structure of matter would overcome the dualism of matter and field. In his account, Corry emphasizes that Hilbert received Mie's theory through the mediation of Max Born, who had cast it into a much more compact and lucid form. Einstein's "Outline" theory of gravitation, on the other hand, was a precursor of his final theory of general relativity, with virtually all characteristics of the latter except for the fact that its gravitational field equations were not generally covariant. Specifically, Einstein had already taken the step from a scalar to a tensorial theory by introducing the crucial concept of the metric tensor to represent the gravitoinertial field. In a comprehensive exposition of this theory, Einstein had taken some pains to cast the theory into a mathematically well-elaborated framework, using concepts of tensor calculus as developed by Grigorio Ricci and Tullio Levi-Civita, and had included a formal derivation of its field equations that also implied a proof of the alleged necessity of their restricted covariance.

The story of the discovery of the correct field equations unfolded after Einstein, at Hilbert's invitation, visited Göttingen in July 1915 to present a series of six Wolfskehl lectures on his generalized theory of relativity to Hilbert and his colleagues. Corry reminds us that this visit took place in the midst of World War I, and he carefully reconstructs the list of participants at Einstein's lectures from among those who would not have been away fighting in the trenches. He then gives a detailed chronology of the course of events that resulted in the almost simultaneous publication of Einstein's and Hilbert's respective papers. Corry emphasizes the differences rather than similarities between the page proofs for Hilbert's communication and its final published version, which leads him to underestimate, I think, Hilbert's achievements. It is true, as had been pointed out in the 1997 paper, that the proofs differ in some significant respects from the printed version, imposing a third axiom that still restricted the general covariance of the theory. They also did not yet contain the Einstein equations in their explicit form. But after all, it was Hilbert who quickly and independently realized that the mathematical representation of Einstein's outline theory was flawed and who, putting his knowledge of variational calculus and invariant theory to work, advanced an original synthesis of Mie's and Einstein's theories and first presented the final gravitational field equations in a variational formulation that is still rightly associated with his name.

Corry concludes his account with a semifinal chapter on Hilbert's work on the theory of general relativity in the years 1916–1918. He finishes his story with brief remarks on Hilbert's work on quantum theory and some concluding comments on the issue of the priority debate between Einstein and Hilbert, on the relationship between general relativity and geometry, and on Hilbert's contribution to the emergence of the theory of general relativity.

As far as it goes, this book is an enormously useful, smoothly written, and well-informed account of Hilbert's involvement with the physical sciences, based on a reliable and broad knowledge of the pertinent literature. It ends, somewhat artificially, with the publication of Hilbert's First Communication on the Foundations of Physics of late 1915 and its immediate aftermath. The final comments on Hilbert's concerns with quantum theory are really only a place holder for a similar study that remains to be written. Hilbert's Second Communication on the Foundations of Physics, published in late 1916, is only discussed peripherally, as is Hilbert's involvement in the intense and interesting debate around the issue of energy–momentum conservation in general relativity. Parties to this debate, in addition to Einstein and Hilbert, also included Felix Klein and Emmy Noether and led to the latter's famous theorems, a special case of which had already been formulated by Hilbert in his First Communication. I was surprised to find that Hilbert's lecture on axiomatic thought, delivered to the Swiss Mathematical Society in September 1917, was omitted from the chronology of events assembled in one of the appendices and was discussed only in passing on two pages in the penultimate chapter. Hilbert's tour d'horizon in this lecture on the method of axiomatization in various fields of pure and applied mathematics, as well as in the natural sciences, would have provided an ideal counterpoint, I believe, to a discussion of Hilbert's 1905 lecture course. More than once, in fact, Corry's account has left me frustrated with an almost characteristic vagueness and inconclusiveness. Despite the richness of the historical documents that he assembles before our eyes and in spite of all the immensely valuable information about the historical context in which Hilbert operated, Corry rarely confronts the reader with precise and definite claims. In his concluding sections, it seems to me that Corry fails to pull together as many open strands of his rich account as he might have. Instead of stating clearly Hilbert's characteristic perception of, or contribution to, the issues under consideration, Corry slips back into a received historical assessment of the emergence of general relativity that is biased by our fixation on Einstein's life and work. It is inevitable then that Hilbert, according to Corry, "came to gradually abandon his own idiosyncratic path and eventually join the mainstream wholeheartedly" (p. 404) and that "his theory not only proved to be physically untenable, but also was far from Hilbert's own stringent mathematical demands and work habits" (p. 437). It appears to me that the author arrives at such conclusions against his intentions and adduced evidence, and, alas, so much more could be said if we could only take on a different perspective, one for which Corry himself has put together a tremendous amount of support.

But with a meritorious book of this caliber one should not end with a complaint about missed opportunities. Corry's account of Hilbert's concerns with the natural sciences refreshingly reminds us that the history of theoretical physics and mathematics is not anemic but full of surprising details that challenge our traditional image of an all too simple and linear development of ideas. The book is a must-read for everyone interested in Hilbert, in the history of 20th-century mathematics, and especially in the pervasive tension between pure and applied mathematics, or in the complex relation between mathematics and physics. The rich historical background that Corry provides will be of great value for future discussions of a variety of open issues in these fields.

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Tilman Sauer
Einstein Papers Project,
California Institute of Technology 20-7,
1200 E. California Blvd., Pasadena, CA 91125, USA
E-mail address: tilman@einstein.caltech.edu

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"zwei wirkliche Kerle": Neues zur Entdeckung der Gravitationsgleichungen der Allgemeinen Relativitätstheorie durch Albert Einstein und David Hilbert
By Daniela Wuensch. Göttingen (Termessos). 2005. ISBN 3-938016-04-3, 126 pp.

The book under review is presented as a case study in how formal mathematization lends clarity to a physical theory. Surely it would be hard to think of a more promising field in which to explore that kind of interplay than Einstein's theory of general relativity, which became a virtual beehive for mathematical activity after 1915. And, just as surely, Göttingen makes an excellent locale for the focus of such an investigation. Yet one senses from the beginning that here this stated purpose is mere window dressing and an essentially empty promise. For although the topic of mathematization rears its head in a few places in this book, it quickly emerges that the author has another, far more pressing agenda, namely to resolve a "priority dispute." Daniela Wuensch wants to decide who was the first to obtain the famous gravitational field equations of general relativity, Einstein or Hilbert. (Hilbert? Right!) Once we