MR1391720 (97i:01023) 01A60 00A30 00A35 01A55 03A05 03B30 18-03
Corry, Leo (IL-TLAV-HS)

★**Modern algebra and the rise of mathematical structures. (English summary)**
Science Networks. Historical Studies, 17.
Birkhäuser Verlag, Basel, 1996. xiv+460 *pp.* $139.00. *ISBN* 3-7643-5311-2

The concept of mathematical structures is often used to describe the development of mathematics in the 20th century. But, as Corry points out, the idea of a mathematical structure is an idea whose nature, meaning and role are not clearly understood and have seldom been discussed systematically. The author tries to answer some of the questions which arise in connection with the emergence of mathematical structures. To do this, he examines the problem in a much broader context and makes a distinction between the body of knowledge and the images of knowledge. According to his definition the former includes statements that are answers to questions related to the subject matter of the given discipline, theories, facts, methods and open problems, and the latter includes the claims which express knowledge about the discipline qua discipline. Both body and images are organically interconnected and Corry compares their relationship with that between a text and its context.

The book is divided into two parts and the headings of the chapters are the following: Introduction: Structures in mathematics. Part One: Structures in the images of mathematics. 1. Structures in algebra: changing images; 2. Richard Dedekind: numbers and ideals; 3. David Hilbert: algebra and axiomatics; 4. Concrete and abstract: numbers, polynomials, rings; 5. Emmy Noether: ideals and structures. Part Two: Structures in the body of mathematics. 6. Oystein Ore: algebraic structures; 7. Nicolas Bourbaki: theory of structures; 8. Category theory: early stages; 9. Categories and images of mathematics.

As seen from the contents the author chose ideal theory to exemplify his ideas. In the first part of the book he describes the rise of ideal theory from its beginnings in the work of R. Dedekind up to those of Emmy Noether. Furthermore, he explains what he understands by the structural approach to algebra. Therefore he compares the presentation of algebraic knowledge in the textbooks at the end of the 19th century and in the classic book by B. L. van der Waerden [*Moderne Algebra. Parts I and II*, G. E. Stechert and Co., New York, 1943; MR0009016 (5,88a)].

In Part Two various mathematical theories are discussed which have emerged since the mid-1930s. First, Corry presents Ore's attempt to develop a general concept of structure which he did between about 1935 and 1945. Secondly, he analyses the theory of structures of Bourbaki. Thus, he demonstrates for instance that structures did not play an important role in the "Eléments de mathématique" and that the notion of mother structures as well as the picture of mathematics as a hierarchy of structures appeared above all in popular articles about mathematics. Thirdly, he describes the theory of categories, which is characterized as the most elaborate and successful instance of an axiomatized theory allowing for a systematic characterization and analysis of the different structures, and the recurring mathematical phenomena that arise in them. In particular, the relations between the theory of categories and Ore's theory, Bourbaki's concept of structure as well as universal algebra are considered. In each chapter the author tries hard to distinguish carefully between the various meanings of the concept of structure in each of these theories and to expound the reasons for their genesis.

The book concludes with some theoretical reflections on the extent to which each of these theories gave a correct representation of the idea of mathematical structure and influenced changed images of mathematical disciplines. Nevertheless, all these attempts have so far failed to provide a thorough explanation of the organization of mathematics in terms of structures.

The book is an inspiring mixture of history of mathematics and theoretical reflections about the rise of mathematical structures. Moreover, it offers an interesting, broad and thorough discussion of the concept of structure which is worth reading for everyone who is concerned with the history of 20th-century mathematics. A detailed bibliography of 35 pages, a subject- and an author-index complement the book. *Karl-Heinz Schlote*